

THE METROPOLITAN MUSEUM OF ART

Papers

ON THE
RATIONALIZATION
OF SIGHT

*With an Examination of
Three Renaissance Texts on Perspective*

BY WILLIAM M. IVINS, JR.

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OF THREE RENAISSANCE TEXTS
ON PERSPECTIVE

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NOTE

The "pathetic fallacy" of the following essays is this:

One night in the late spring of 1936 the writer innocently thought to kill the short end of a hot evening by drafting an even shorter footnote to a book on renaissance book illustration. Before morning he had spent hours madly drawing lines with a ruler and was entangled in a subject that, occupying much of his subsequent leisure time, has flung him headlong and bewildered into fields and problems the very existence of which was then unknown to him and of which he now knows little more. On trains and boats that subject has provided him with the best of crossword puzzles, and in hours of lonesomeness and worry with the most efficient and cleansing of diversions.

The following essays are but stumbling and tentative drafts of small parts of that proposed footnote. However, should they direct the attention of a few students of art to some of the problems they lead up to, their publication in their present state will have been justified.

ON THE RATIONALIZATION OF SIGHT

FOR SOME TIME past the present writer has been pursued by the notion that the most important thing that happened during the Renaissance was the emergence of the ideas that led to the rationalization of sight. This is a matter so different from the fall of Constantinople, the invention of printing from movable types, the discovery of America, the Reformation or the Counter Reformation, or any of the other traditional great events of that period, that a hasty account of it seems excusable before embarking upon a detailed examination of the mechanics of the perspective schemes of Alberti, Pelerin (known as the Viator), and Dürer, in which the effort towards that rationalization received its first expression in Italy, France, and Germany.

In order to have ideas about the returns given us about nature by our five senses, it is necessary to have some system of symbols by which to represent those returns and some grammar or rule by which those symbols are given logical relationships. Lacking such symbols, or a grammar for their use, the task of thinking becomes too onerous to be carried very far. A symbol that cannot be exactly duplicated, or, what comes to the same thing, a symbol that of necessity undergoes fortuitous changes of meaning in the course of repetition or duplication, is of very limited usefulness. A system of symbols without logical schemes, both for its interrelations and combinations within itself and, if it symbolize external fact, for its two-way, or reciprocal, correspondence with that external fact, is also of very limited usefulness. However interesting or important such symbols or series of symbols may be for personal intuition they obviously

have little or no value for rationalization.

Words as symbols have no meanings except such as they get from general convention or specific agreement as coupled with recognitions arising through concrete experience, and thus are incapable of conveying information about unique characteristics to people who are not acquainted with those characteristics at first hand. It is doubtful whether a recognizable portrait has ever been painted from a verbal description. As yet no symbolization except a very poor verbal tautology has ever been worked out for the returns given us by the senses of taste and smell. At most we can say that strawberries taste like strawberries and that roses smell like roses. The symbolization for the returns given us by the sense of hearing is extremely limited and as yet has proved incompetent to deal with sounds that lie outside the conventional restrictions of the notation of words and music. There is no symbolism that has been able to record or deal with such things as the personal timbre or characteristics of a human voice. The phonograph and the sound cinema, which after all are very recent inventions, while providing a method for the duplication of sounds, provide no symbols for them and no grammar or rules for the combination of such symbols.

The ancient Greeks worked out a highly abstract symbolism for certain very elementary and limited space intuitions and provided it with a most remarkable grammar. The two together are known to us as Euclidean geometry, the origin of which in tactile-muscular intuition is shown by its nearly complete preoccupation with metrical problems and its essential dependence upon congruence. The

dominance of tactile-muscular intuition in Greek geometry and the failure of that geometry to take account of visual intuition is exemplified by the fact pointed out by L. Cremona that "most of the propositions in Euclid's Elements are metrical, and it is not easy to find among them an example of a purely descriptive theorem."¹ Although the Greeks worked all around the problem of perspective, as is shown, for example, by their interest in conics, their knowledge of the anharmonic ratio, and their discovery of such theorems as that about the inscribed hexagon to which Pappus's name is attached,² they seem never to have realized that there was such a thing as a mathematical problem of perspective. The underlying tactile assumptions of Euclidean geometry are excellently exhibited in its basic postulate about parallel lines. If we get our awareness of parallelism through touch, as by running our fingers along a simple molding, there is no question of the sensuous return that parallel lines do not meet. If, however, we get our awareness of parallelism through

sight, as when we look down a long colonnade, there is no doubt about the sensuous return that parallel lines do converge and will meet if they are far enough extended. Although Euclid was well aware of this (see his *Optics*, Theorem VI) and was explicit about the fact that his famous fifth postulate³ was a postulate, it was not until the seventeenth century that for the first time a mathematician adopted convergence at infinity as the basis of a definition of parallel lines.⁴

At the very beginning of human history men discovered in their ability to make pictures a method for symbolization of their visual awarenesses which differs in important respects from any other symbolic method that is known. As distinguished from purely conventional symbols, pictorial symbols can be used to make precise and accurate statements even while themselves transcending definition.⁵ In spite of this, picturemaking long remained a most inefficient sort of symbolization. There were two great reasons for this inefficiency: one, that no picture could be ex-

(1) *Elements of Projective Geometry*, Oxford, third edition, p. 50.)

(2) For these and other similar instances, see Sir Thomas Heath's *A History of Greek Mathematics*, Oxford, 1921, vol. II, pp. 270, 381, 397, 419, 521, etc.

(3) Euclid's fifth postulate is today perhaps best known through Playfair's eighteenth-century equivalent, that only one line may be drawn through a given point parallel to a given line.

(4) If one remembers correctly, it was Ernst Mach who picturesquely pointed out that if men were fastened immovably to rocks like mollusks in the sea they could have no sensory intuition of Euclidean space. F. Enriques, who has discussed the spatial intuitions that come from visual and from tactile-muscular sensations, has said: "F. Klein a le premier remarqué cette difference entre les propriétés descriptives et les propriétés métriques." (*Encyclopédie des sciences mathématiques . . .*, tome III, vol. I, p. 63, and see also Enriques's *Leçons de géométrie projective*, Paris, 1930, p. 3.) The way in which the tactile-muscular habit of thought inhibited the ancient geometers is very remarkably shown by Heath's account (*op. cit.*, vol. II, p. 521) of Peithon and Serenus (fourth century A.D.). This account is so interesting that I quote it

in extenso. "In the propositions (29-33) from this point to the end of the book Serenus deals with what is really an optical problem. It is introduced by a remark about a certain geometer, Peithon by name, who wrote a tract on the subject of parallels. Peithon, not being satisfied with Euclid's treatment of parallels, thought to define parallels by means of an illustration, observing that parallels are such lines as are shown on a wall or a roof by the shadow of a pillar with a light behind it. This definition, it appears, was generally ridiculed; and Serenus seeks to rehabilitate Peithon, who was his friend, by showing that his statement is after all mathematically sound. He therefore proves, with regard to the cylinder, that, if any number of rays from a point outside the cylinder are drawn touching it on both sides, all the rays pass through the sides of a parallelogram (a section of the cylinder parallel to the axis) — Prop. 29 — and if they are produced farther to meet any other plane parallel to that of the parallelogram the points in which they meet the plane will lie on two parallel lines (Prop. 30); he adds that the lines will not seem parallel (*vide* Euclid's *Optics*, Prop. 6)."

(5) In thinking about symbols it is necessary to remember that while some symbols are defined by their

actly duplicated, and the other, that there was no rule or grammatical scheme for securing either logical relations within the system of pictorial symbols or a logical two-way, or reciprocal, correspondence between the pictorial representations of the shapes of objects and the locations of those objects in space.⁶

Until the end of the fourteenth century this was the condition of man's ability to symbolize his sensuous awareness of nature. To it may be attributed much of the failure of classical and mediaeval natural science.

At the end of the fourteenth century or the beginning of the fifteenth century someone somewhere in Europe began to make woodcuts. Originally the woodcut was a mere labor-saving device for the quantity production of sacred images. It was the earliest form of the printed picture. By the end of the fifteenth century men were printing pictures from engraved and etched metal plates as well as from wooden blocks. The printing of pictures provided for the first time a technique which made possible the exact duplication of pictorial symbols for visual awarenesses.⁷

The invention of the printed picture was thus not improbably an event unique in the history of European thought. Very shortly

references, other references are defined by their symbols. The more closely a highly organized and purely conceptual subject, such as mathematics, defines its symbols, the wider is the range of variation that may be introduced into the physical forms of the symbols without effecting change in their significance. The more closely symbols (e.g. pictures) define unorganized and concrete subjects, such as the materials of visual sense awarenesses, the narrower is the range of variation that may be introduced into the physical forms of the symbols without effecting change in their significance. Thanks to the pictorial symbol's sensuously immediate definition of its reference, it is basic for many of the recognitions of similarity which must be made before practical knowledge or science is possible.

(6) The only pre-Renaissance statement I have found to show that people were ever specifically aware of the difficulties of a pictorial symbolism that was not accu-

after it happened there was another unprecedented event which, coming in similar fashion to a society that was not prepared for it, took a long time before its mechanics were understood or its implications were recognized. This was Leone Battista Alberti's discovery of a simple but logical scheme for pictorial perspective.

Perspective may be regarded as a practical means for securing a rigorous two-way, or reciprocal, metrical relationship between the shapes of objects as definitely located in space and their pictorial representations. Important as this is to picturemaking in the narrowest sense, it is doubtless even more important to general thought, because the premises on which it is based are implicit in every statement made with its aid. Either the exterior relations of objects, such as their forms for visual awareness, change with their shifts in location, or else their interior relations do. If the latter were the case there could be neither homogeneity of space nor uniformity of nature, and science and technology as now conceived would necessarily cease to exist. Thus perspective, because of its logical recognition of internal invariances through all the transformations produced by changes in spatial lo-

repeatably repeatable is contained in chapters 4, 5, and 10 of Book xxv of Pliny's *Natural History*. Little more interesting or directly to the point can be desired than this account of why some Greek botanists gave up the attempt to illustrate their books.

(7) The history of the graphic techniques is neither more nor less than the history (1) of the extension of the ability exactly to duplicate the symbols of visual awareness and (2) of the extension of the power of those symbols sensuously to define unique personal characteristics that transcend purely formal or conventional notation. The historians of "fine prints," because of their limited technical approach and also because of their preoccupation with primitive rarities and the very occasional artistic masterpieces, have with remarkable unanimity disregarded both the expansion of the social utility of the graphic media and their functional growth and intellectual importance as tools of knowledge and thought.

cation, may be regarded as the application to pictorial purposes of the two basic assumptions underlying all the great scientific generalizations, or laws of nature.⁸

Alberti's perspective scheme of 1435-1436 (see p. 14 below) marked the effectual beginning of the substitution of visual for tactile space awareness, because its novel procedure of central projection and section⁹ not only automatically brought parallel lines together in logically determinable vanishing points, but provided a basis for the hitherto missing grammar or rules for securing both logical relations within the system of symbols employed and a reciprocal, or two-way, metrical correspondence between the pictorial representations of objects and the shapes of those objects as located in space.

Ever since Alberti made his statement, men have been busy, some misunderstanding and some developing it. Leonardo da Vinci and others who understood it reduced it to a form, known as the "costruzione legittima," that was practical for artists.¹⁰ Viator published the variant which is now known in the studios as three-point perspective in his *De artificiali perspectiva* of 1505. Dürer, whose *Unterweysung der Messung* was published in 1525, was acquainted with the method of projection and section, but failed to understand it, as apparently did his immediate German successors. Vignola, in the first half of the sixteenth century, taught both the *costruzione legittima* and the three-point method, but the substance of his teaching was not published until Eg-

(8) Cf. B. A. W. Russell, *An Essay on the Foundations of Geometry*, Cambridge, 1897, *passim*.

(9) The late Greek geometers on rare occasion utilized this procedure, as for example in propositions 28 and 29 of the fourth book of Pappus's *Synagoge* (see Heath, *op. cit.*, vol. II, p. 380), but would seem never to have realized its possibilities or to have developed it.

(10) The text of leaf 42 recto of Leonardo's Manuscript A (see reproduction, p. 23 below) proves conclusively

natio Danti's *Le due regole della prospettiva pratica* appeared at Rome in 1583. Guidobaldo del Monte in his prolix *Montis perspektivae libri sex* of 1600 summed up the perspective knowledge of the sixteenth century and worked out a number of elaborate variations but seemingly added little to the basic theory. He is said to have been the first to use the phrase vanishing point ("punctum concursus").

Kepler's postulation, in his *Ad vitellionem paralipomena* of 1604, that parallel lines meet at a point at infinity¹¹ was the independent mathematical recognition of an operational fact implicit in Alberti's construction and indirectly stated by him in his text (see p. 22 below). It has been said that Kepler's postulation marks off modern from classical geometry.

It was not, however, until the 1630's that for the first time a mathematician of genius attacked the specific problem of perspective. This man, Girard Desargues of Lyons, the greatly admired friend of Descartes and Fermat, opened the way to both the perspective and the descriptive geometries. That Alberti preceded Kepler by one hundred and seventy years, and Desargues by two hundred years, throws much light upon the mathematical knowledge and ability of the fifteenth and sixteenth centuries. Among many other things, Desargues discovered the theorem about perspective triangles now known by his name, and, from purely perspective considerations, he postulated in so many words that parallel

that Leonardo was fully aware of the strict two-way metrical correspondence between a correctly made perspective drawing of an object in space and the object itself.

(11) See Charles Taylor's article "Geometrical Continuity" in *Encyclopaedia Britannica*, eleventh edition. The essential passage from Kepler's text is reprinted in H. F. Baker's *Principles of Geometry*, Cambridge, 1929, vol. I, p. 178.

lines in a plane meet at a point at infinity.¹² In 1640, the year after Desargues's *Brouillon proiect d'une atteinte*, his pupil, Blaise Pascal, by the use of its methods, worked out the theorem about the hexagon inscribed in a conic. Thus Desargues and Pascal, between them, developed the two basic theorems of the modern geometry of perspective.¹³ Those who think of perspective only as a more or less unimportant subject in the curriculum of an art school should find food for thought in the facts that Desargues is reputed to have been the first to design an epicycloidal gearing,¹⁴ and that every engineering and architectural school now requires that its students have a knowledge of descriptive geometry.

Because of the scattered way in which Desargues published his results — his very important *Brouillon proiect d'une atteinte* was lost

(12) Desargues, writing in 1636, said: "Quand les lignes suiet sont paralelles entr'elles, & que la ligne de l'oeil menée paralelle à icelles, n'est pas paralelle au tableau; les aparences de ces lignes suiet, sont des lignes qui tendent toutes au point auquel cette ligne de l'oeil rencontre le tableau, d'autant que chacune de ces lignes suiet est en un mesme plan avec cette ligne de l'oeil, en laquelle tous ces plans s'entre-coupent ainsi qu'en leur commun essieu, & que tous ces plans sont coupez d'un autre mesme plan le tableau." (See A. Bosse, *Manière universelle de Mr Desargues pour pratiquer la perspective*, 1648, p. 333.) If this be compared with Alberti's construction (see p. 22 below) it will be seen to be a verbal statement of what happens in the operation of that construction. In 1639 Desargues, in his *Brouillon proiect d'une atteinte . . .*, said: "Pour donner à entendre l'espece de position d'entre plusieurs droites en laquelle elles sont toutes paralelles entr'elles, il est icy dit que toutes ces droites sont entr'elles d'une mesme ordonnance, dont le but est à distance infinie, en chacune d'une part et d'autre." (See Poudra's *Oeuvres de Desargues*, Paris, 1864, vol. 1, p. 104.) Desargues's 1636 demonstration of his theorem about perspective triangles will be found at p. 340 of the book by Bosse cited above, as well as in Poudra's edition of Desargues's *Oeuvres*.

(13) H. Wiener, in his "Ueber Grundlagen und Aufbau der Geometrie" (*Jahresber. d. deutsch. Math. Verein*, vol. 1, 1892, p. 47) says: "Diese beiden Schliessungssätze [i.e. the theorem of Desargues about perspective triangles and that of Pascal about the particular case in which the

hexagon is inscribed within a conic degraded to two straight intersecting lines] aber genügen, um ohne weitere Stetigkeitsbetrachtungen oder unendliche Prozesse den Grundsatz der projectiven Geometrie zu beweisen, und damit die ganze lineare projective Geometrie der Ebene zu entwickeln."

hexagon is inscribed within a conic degraded to two straight intersecting lines] aber genügen, um ohne weitere Stetigkeitsbetrachtungen oder unendliche Prozesse den Grundsatz der projectiven Geometrie zu beweisen, und damit die ganze lineare projective Geometrie der Ebene zu entwickeln."

(14) See Chasles's *Aperçu historique sur l'origine et le développement des méthodes en géométrie*, Paris, third edition, p. 86.

(15) "Monge en conçut les idées fondamentales vers 1775, il les élaborâ lentement et les exposa pour la première fois d'une façon systématique à l'École Normale, an III de la République. Mais il ne fut autorisé à publier ses importantes découvertes que l'an VII, à cause de la crainte éprouvée par le Gouvernement que les étrangers n'en tirent profit pour leurs ouvrages de défense militaires." (M. Solovine, at p. x of the Notice biographique prefacing his edition of Monge's *Géométrie descriptive*, Paris, 1922.) The following sentences from the short "Programme" which Monge himself prefixed to his book are not without interest: "Cet art a deux objets principaux. Le premier est de représenter avec exactitude, sur des dessins qui n'ont que deux dimensions, les objets qui en ont trois, et qui sont susceptibles de définition rigoureuse. Sous ce point de vue, c'est une langue nécessaire à l'homme de génie qui conçoit un projet, à ceux qui doivent en diriger l'exécution, et enfin aux artistes qui doivent eux-mêmes en exécuter les différentes parties. Le second objet de la Géométrie descriptive est de déduire de la description exacte des corps tout ce qui suit néces-

figures: *Ouvrage utile à ceux qui s'occupent des applications de la géométrie descriptive et d'opérations géométriques sur le terrain*, in which projective geometry was finally developed into a full-fledged mathematical discipline.¹⁶ In 1847 von Staudt freed perspective geometry of metrical notions.¹⁷ The development at the hands of subsequent workers has been most remarkable, especially as leading up to the study of the foundations of geometry. Methods have been discovered by which Euclidean geometry and the various non-Euclidean geometries have been so related to projective geometry that Cayley felt justified in his enthusiastic statement that "projective geometry is all geometry."¹⁸

On the immediately practical side it is hardly too much to say that without the development of perspective into descriptive geometry by Monge and into perspective geometry by Poncelet and his successors modern engineering and especially modern machinery could not exist. Many reasons are assigned for the mechanization of life and industry during the nineteenth century, but the mathematical development of perspective was absolutely prerequisite to it. Professor A. N. Whitehead has somewhere remarked that the great invention of the nineteenth century was that of the technique of making inventions. The inventions of Monge and Poncelet were among the most important of the intellectual

sairement de leurs formes et de leurs positions respectives. . . . On contribuera donc à donner à l'éducation nationale une direction avantageuse, en familiarisant nos jeunes artistes avec l'application de la Géométrie descriptive aux constructions graphiques qui sont nécessaires au plus grand nombre des arts, et en faisant usage de cette Géométrie pour la représentation et la détermination des éléments des machines. . . ."

(16) There are few stories more romantically interesting or intellectually suggestive than those of the early lives of Monge and Poncelet. Poncelet, captured by the Russians during Napoleon's retreat from Moscow, and languishing in prison for several years without books or

tools which made that great invention possible.

It is interesting to notice that, just as the earliest datable European prints were made during the lifetime of Alberti (1404-1472), so Monge (1746-1818) and Poncelet (1788-1867) were contemporaries of Niépce (1765-1833) and Fox Talbot (1800-1877), to whose ingenuity we owe the first photography, a form of picturemaking that is not only precisely duplicable but one in which geometrical perspective is so inherent that today the camera is used as a surveying and measuring instrument¹⁹ as well as a tool for the making of precisely duplicable pictures of unique characteristics that transcend notation in terms of convention, for instance, in its use in the attributions of connoisseurship. Photographic pictures have entered so deeply into the consciousness of Western Europe and America that now there are few people who are not unhappy with a modern picture that is too obviously out of photographic perspective.

The most marked characteristics of European pictorial representation since the fourteenth century have been on the one hand its steadily increasing naturalism and on the other its purely schematic and logical extensions. It is submitted that both are due in largest part to the development and pervasion of methods which have provided symbols, repeatable in invariant form, for representation

papers, preserved his sanity during his enforced idleness by making some of the greatest of all mathematical discoveries.

(17) In the preface to his *Geometrie der Lage* of 1847, he said: "Ich habe in dieser Schrift versucht, die Geometrie der Lage zu einer selbständigen Wissenschaft zu machen, welche des Messens nicht bedarf."

(18) Compare the remark made by Jean Nicod, *Foundations of Geometry & Induction*, London, 1930, p. 182: "The order of views thus becomes the only fundamental space of nature."

(19) See, e.g., H. Deneux, *La Métrophotographie . . .*, Paris, 1930.

of visual awareneses, and a grammar of perspective which made it possible to establish logical relations not only within the system of symbols but between that system and the forms and locations of the objects that it symbolizes.

In the middle sixteenth century Brunfels and Fuchs issued the first botanies provided with printed illustrations adequate to the symbolization of the unique characteristics of the various plants and flowers. In 1543 Vesalius and John of Calcar produced the first fully illustrated anatomy, that is, the first grammar of the human figure which, naming the various bones, muscles, etc., defined them by exact reference to pictures, which, being printed from unchanging wooden blocks, remained invariant throughout the entire edition. Since that time, thanks in important measure to the availability of methods for the exact duplication of logically arranged pictorial symbols for visual awareneses, scientific description has proceeded at a constantly accelerating rate. Scientific classification, which was practically impossible for many things so long as such methods were not available, has now because of them made enormous strides. Those methods have perhaps reached some of their most popularly acclaimed achievements in classification in the fields of archaeology, artistic connoisseurship, medical diagnosis, and criminal detection, knowledges and practices that have been completely refashioned since the development of photography and its related processes. Today there are few sciences or technologies that are not predicated in one way or another upon this power of invariant pictorial symbolization.

The constant extensions of the fields of use-

fulness of the pictorial symbol that is precisely duplicable and of the grammars of its use have had a most astonishing effect not only upon knowledge but upon thought and its basic assumptions or intuitions. Where the dominant Greek and mediaeval idea of "matter" seems to have been based on tactile and muscular intuitions, the modern one to a very great extent is based upon visual habits and intuitions. Relativity, which now in one form or another runs throughout contemporary thought and practice, is in large measure a development of ideas that were evolved through the study and use of projective transformations.

From being an avenue of sensuous awareness for what people, lacking adequate symbols and adequate grammars and techniques for their use, regarded as "secondary qualities," sight has today become the principal avenue of the sensuous awareneses upon which systematic thought about nature is based. Science and technology have advanced in more than direct ratio to the ability of men to contrive methods by which phenomena which otherwise could be known only through the senses of touch, hearing, taste, and smell, have been brought within the range of visual recognition and measurement and thus become subject to that logical symbolization without which rational thought and analysis are impossible.²⁰ The discovery of the early forms of these grammars and techniques constitutes that beginning of the rationalization of sight which, it is submitted, was the most important event of the Renaissance.

(20) Nicod, *op. cit.*, p. 172, speaks of "our so-called visual distance which alone is correct enough for science."

THREE RENAISSANCE TEXTS ON PERSPECTIVE

I.

BY COMMON AGREEMENT the three outstanding renaissance texts on perspective are those of Alberti, Viator, and Dürer. Alberti's book, the *Della pittura libri tre*, written in 1435-1436, is generally acknowledged to be the earliest statement of a logically coherent and pictorially adequate scheme of perspective representation. The construction worked out by Alberti, and used after him by generations of Italian artists, is currently known as the "costruzione legittima." Viator's book, the *De artificiali perspectiva*, published at Toul in 1505 and pirated at Nuremberg in 1509, contains the first statement of the familiar "three point" or "distance" method. Dürer's book, the *Unterweysung der Messung*, first published in 1525, was for several generations the most advanced German authority on the subject.

The simplest form of the perspective problem is how to throw a square into a geometrically logical projection. As this is a problem in geometrical optics, and not in physiological optics or psychology, its solution may be regarded as a convention, but a convention of

(1) The page references following my quotations of Alberti's text are to the reprint of the original as given in Hubert Janitschek's edition of *Leone Battista Alberti's Kleinere Kunsttheoretische Schriften*. In making my English versions I have made hard use of the Italian-English dictionaries of Florio (London, 1611) and Hoare (Cambridge, 1925) and even more of the model represented in my illustrations. I have also consulted the translations, into German by Janitschek, into French by Popelin, and by Bartoli and by Domenichi into Italian from a Latin version. In so far as they deal with Alberti's perspective the first three of these translations with their diagrams are ingenious misrepresentations of Alberti's thought. The precedent thus set has been followed by many of the later writers on renaissance perspective. Janitschek's great merit is that he made the original Italian text available.

such great utility and so exceedingly familiar that for practical purposes it has the standing of a "reality." The two different methods of Alberti and Viator produce identical results, as can be proven by elementary geometrical reasoning. In diagrammatic form the two constructions are as shown in figures 1 and 2. BC is the near side of the square to be projected. The vanishing point, A, is anywhere above BC, and as high above it as the observer's eye is above the plane of the square. The projected right and left sides of the square lie along CA and BA. DA is parallel to BC. In Alberti's system a perpendicular is erected through B, cutting DA at E. The distance between the points D and E in Alberti's system, and between the points D and A in Viator's system, is equal to the distance between the near edge of the square and the observer. In Alberti's system the projection of the fourth side of the square is determined by where the line DC cuts the perpendicular BE. In Viator's system it is determined by where the line DC cuts the line BA. It is interesting to note that the point A can be located anywhere along the line DA and does not have to be centered above the points C and B, and that because of this fact the *costruzione legittima* and the distance construction have an ostensible exact similarity when the lines BA and BE happen to coincide.

II.

The first thing we must do, would we understand Alberti's rather obscure text,¹ is rapidly to run through it in the hope of arriving at some idea of what his tools and contrivances were and what it was in particular that he was

trying to do with them. This is especially necessary because it is obvious that Alberti, in writing his descriptions, was not thinking wholly in terms of the problems of the ordinary maker of pictures.

In the second paragraph of the third of his Three Books, Alberti says that *the task of the painter is to represent with lines and color with pigments the visible surface of any object upon any given panel or wall in such fashion that, at a certain distance and in a certain position from the center of vision, it may appear as*

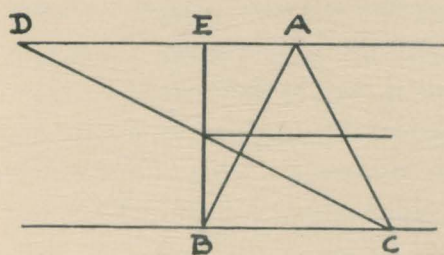


FIG. 1.
ALBERTI'S CONSTRUCTION

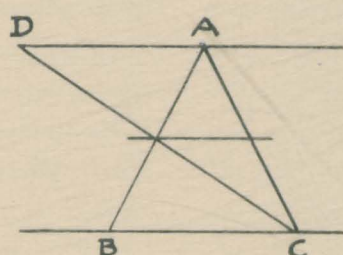


FIG. 2.
VIATOR'S CONSTRUCTION

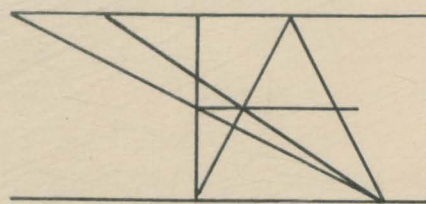


FIG. 3. THE TWO CONSTRUCTIONS
SUPERIMPOSED

*though in the round and will closely resemble the object.*²

In his first book, while describing the lines of vision between the eye and the things it sees, Alberti says: *We may imagine the [visual] rays as though they were very fine threads tightly bound together in a bunch as by an iron band within the eye . . . almost like a pollard of all the rays, the node of which shoots its young branches straight and fine against any opposing surface.*³ He also says that the rays from the eye to the outward boundaries of the field of vision make what is called the pyramid of vision⁴; that when a painter makes a picture of something he sees it is as if his panel were of transparent glass cutting across the pyramid of vision at a given distance from the eye,⁵ and that because of this whoever looks at a picture looks at a cross section of a pyramid of vision.⁶

In his remarks about light, shadows, and reflections, he says that there is much more to be said about those subjects, as was shown by the miraculous pictures ("miracoli della pittura," p. 67) he had made at Rome. Later he says that no picture can resemble nature unless it is seen at a definite distance,⁷ and that he will give the proof of this if he ever comes to write up those "demonstrations" which he had made and which astonished his friends as though they were miracles.⁸ A little later on, at a crucial point in the description of his op-

eration he says that *he determines the distance*

(2) "Dico l'ufficio del pictore essere così: descrivere con linea et tigniere con colori, in qual sia datoli tavola o parete simile vedute superficie di qualunque corpo, che quelle ad una certa distanza et ad una certa positione di centro pajano rilevate et molto simili avere i corpi." — p. 143.

(3) "Et noi qui immaginiamo i razzi quasi essere fili sottilissimi da uno capo quasi come una mappa molto strettissimi legati dentro all'occhio . . . quasi come troncho di tutti i razzi, quel nodo extenda dritissimi et sottilissimi suoi virgulti per sino alla opposita superficie." — p. 57.

(4) "Et questi razzi extrinseci . . . fanno, quanto si dice, quella piramide visiva." — p. 61.

(5) "Se non che in questa superficie si presentino le forme delle cose vedute, non altrimenti, che se essa fusse di vetro tralucente, tale che la piramide visiva indi trapassasse, posto una certa distantia." — p. 69.

(6) "Chi mira una pictura, vede certa intersegatione d'una piramide." — p. 69.

(7) "Cosa niuna dipinta mai parra pari alle vere, dove non sia certa distantia a vederle." — p. 81.

(8) "Ma di questo diremone sue ragione, se mai scriveremo di quelle dimostrazioni quali fatte da noi li amici veggendole et maravigliandosi chiamavano miracoli." — p. 81.

he wants between the eye and the picture⁹ — a thing that no ordinary painter ever thinks about, or is required to.

The clue to the sense of these remarks is given in the *Vita anonyma* of Alberti (I translate from Janitschek's German, p. 229) which says that Alberti "wrote several books on painting, for with the aid of this art he brought about things unheard of and that the spectators found unbelievable, and he showed these things through a tiny opening that was made in a little closed box. . . . He called these things 'demonstrations,' and they were of such a kind that both artists and laymen questioned whether they saw painted things or natural things themselves."

All these hints point to the strong probability that Alberti conducted his researches with a peep show or a visual model. This being so, before we examine his text critically in detail, let us see what the simplest kind of a peep show or model can be, and find out whether we can use it as a means of arriving at a perspective construction or a diagrammatic rule of thumb for doing easy perspective.

The simplest kind of a model is an oblong box with an eye, or peephole, towards the top of one end, an object on the floor or bottom of the box, and a slide or slides which can be inserted perpendicularly in the box between the hole and the object. On each of the slides such a picture of the object can be painted that when it is in its proper position in the box a person looking through the peephole will find it difficult to tell whether he sees the picture on the slide or the object on the floor. The major problem that faces the maker or operator of such a peep show or model is to determine the shape and size of the picture of the

(9) "Poi constituisco quanto io voglia distantia dall'occhio alla pictura." — p. 83.

object to be painted on any given slide, and the position of that picture on the slide. For experimental purposes it is well to use the simplest kind of an object. The best object for an experiment of this kind is a checkerboard, placed on the bottom of the box with its edges parallel to the sides of the box. Once the checkerboard has been placed in position, the next step is to stretch strings from the eye-hole to the intersections of the lines on the checkerboard, to represent the lines of vision between the eye and the checkerboard. (See fig. 6, where for clarity's sake strings have been stretched only to the intersections along two adjoining sides of the checkerboard.)

By stretching strings in this way we are enabled not only to make the lines of our vision visible in a fixed position, but after a fashion to leave them there so that we can walk around and examine them from the sides and top and see what we can discover about their angular and measurable relationships to each other, the eyehole, and the object. The size, shape, and height above the floor of the box, of the various cross sections of the pyramid of vision represented by the strings, can easily be measured at any given point between the eyehole and the checkerboard — a proceeding that is fairly difficult without the strings. It is, however, not operationally necessary to stretch strings from all the intersections on the checkerboard, as the same theoretical and practical results can be achieved by stretching them from alternate intersections along any two adjacent sides of the checkerboard and from the remaining corner — which is what we have actually done with our model.

The easiest way of taking our measurements is by resorting to the age-old trick of the carpenters and stonemasons when they have to get the precise shape of a molding or uneven surface, that is, by cutting a templet. A tem-

plet is merely a piece of thin board one edge of which is gradually and carefully cut away so that when it is finished it will fit exactly up against the molding or uneven surface of which record has to be made. If, then, we cut a templet which will fit over the strings so that it will just touch them as they pass through it and which on either side of the strings will reach down to the bottom of the box, we shall have a way of getting the precise measurements of the cross section of the strings at any point we desire between the eyehole and the checkerboard. The nearer to the checker-

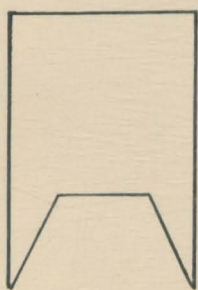


FIG. 4

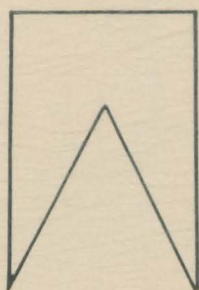


FIG. 5

board the templet is placed the lower and wider the cross section of the strings will be. The nearer to the eyehole it is placed the higher and the smaller the cross section will be. And, if it is placed right at the eyehole the cross section will be merely a small hole or point at the height of the eyehole from the floor of the box. Thus, if the templet is cut to fit the strings somewhere between the eyehole and the checkerboard it will have a shape more or less like figure 4 and can be slid along the box from its calculated position away from the eyehole but not towards the eyehole, because the strings rise up and get in the way. But if the templet is cut in a triangular form like figure 5, with the top corner high enough to fit over the eyehole and the lower corners far enough apart to fit over the strings where they meet the edges of the checkerboard, the

templet can be slid along the box in either direction and will fit the strings anywhere between the eyehole and the checkerboard. With a templet of this shape the heights of the strings can be marked upon the templet as they are at any given position the templet may occupy. We will use this second form of templet (see fig. 7).

In this way, without knowing anything about geometry or perspective, we have succeeded in discovering a means by which we

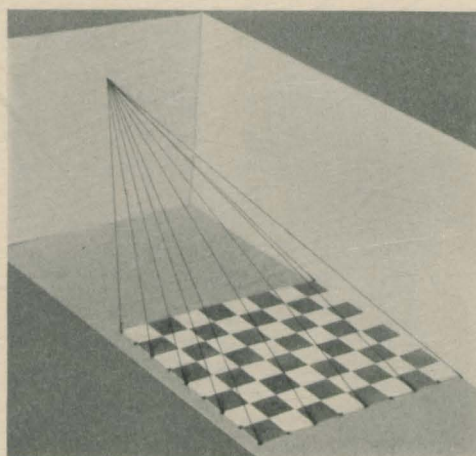


FIG. 6

can draw an accurate picture of the checkerboard as seen in perspective on a slide at any position in our box. The trouble with this method is that it will only work for an object small and simple enough to be placed on the floor of the model and to have strings stretched to it from the eyehole of the model. Therefore our next task is to discover some way by which a diagram can be drawn by rule of thumb which will enable us to cope with the problem of the perspective rendering of objects too big or too far away to be put in a model.

One way of doing this is by fixing the templet in a definite position at the near edge of the checkerboard and then looking at our strings from two different positions. The first

position from which we look at the strings and templet is from the side. From it we see something like figure 7. We mark the heights of the strings on the templet. We then make a simple schematic diagram to scale of what we have seen (but doing it as though we had seen the model from a point on the line passing through the two points of the templet), like figure 8. We then move around to the end of the model at a position exactly opposite the eyehole, and look at the strings again.

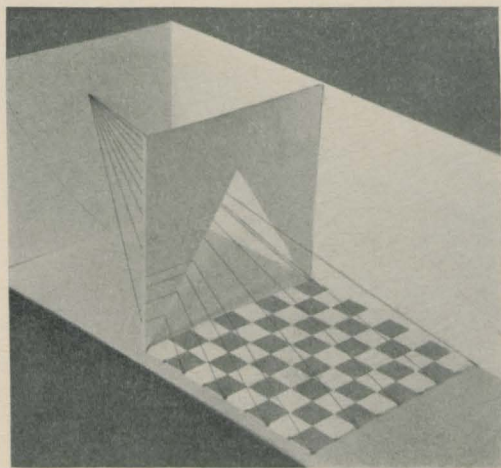


FIG. 7

What we see looks like figure 12. We now make another simple diagram, to the same scale as our first (i.e., fig. 8), of what we have seen, like figure 9, and then by carrying the lines we have marked on the templet across the diagram we have something like figure 10, which is exactly what we have been hoping to find. For if we have the measurements of the checkerboard we want to throw into perspective and know how far away and how much below the eye it is, all we have to do is to make measured diagrams to the same scale from each of our two points of view. One diagram (fig. 8) will give us the apparent heights of the transverse lines on the checkerboard at any given position between our eyes and the checkerboard, and the other diagram will give

us the way in which the orthogonal lines on the checkerboard appear to converge. By carrying our determination of the heights over from our first drawing to our second drawing, we get the picture we want of our checkerboard as seen in perspective. Needless to say, either of these diagrams can be made before the other, and as matter of fact when we come to examine Alberti's text we shall find that he reversed the order in which we have made them.

Before going further, it is well to point out several things about these views and diagrams.

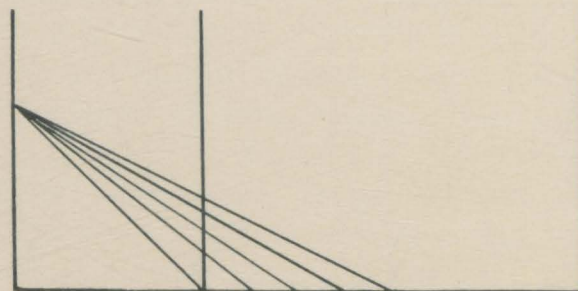


FIG. 8

The most interesting thing about the end elevation of the strings that we made in our second diagram (fig. 9) is that, provided the point of convergence of the strings remains at the same height above the checkerboard and at the same distance from its two orthogonal edges, the diagram remains the same no matter at what angle to right or left the bundle of strings may tilt, as seen in our first diagram (fig. 8). The other thing which it is well to have in mind is that our two views represent the same point of convergence of the strings as seen from different points of view and that the names given to the several representations of that point of convergence in our modern terminology relate not to different things but to different aspects of the one thing as seen from different positions. As most perspective constructions or working diagrams contain

indications of the several aspects of the one thing as seen from several points of view it is most convenient to have different names for its different aspects. Alberti called the aspect of the point of convergence shown in the second of our two views the center point, but he used no name for the aspect shown in our first view, which in modern terminology is known as the distance point.

A simpler method, which requires but one drawing and has other very great advantages, could well have been arrived at in the following manner: One day we go to our box to

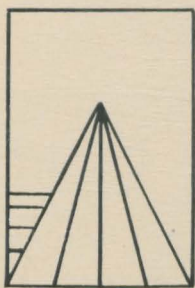


FIG. 9



FIG. 10

make some adjustments in it, and in order to do this we take the triangular templet off the strings, and, temporarily to get it out of our way, we lean it up against the side of the box with its lower angles or points quite accidentally somewhere near the edges of the checkerboard. At the time we do this the templet happens to have marked on it the heights of the strings as they were when the templet was in position directly on the edge of the checkerboard nearest to the eyehole. When we have finished our adjustments and turn to pick up the templet in order to put it back in place over the strings, we notice something we had not seen before which makes us place the templet flat against the side of the box with its lower corners directly in contact with the corners of the checkerboard, so that it looks like figure 11. As soon as we have done

this we see that the perpendicular edge of the templet nearest the eyehole marks the position which the templet had when we marked the height of the strings upon it and that the strings cut across that perpendicular edge exactly at the marks we had made on the templet to indicate the heights of the strings. We then try the templet in various positions over

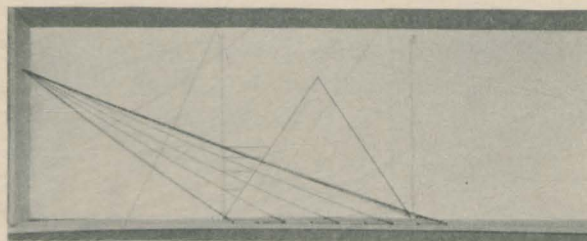


FIG. 11

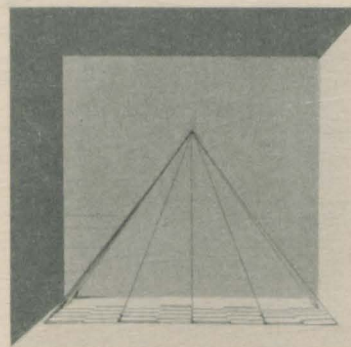


FIG. 12

the strings between the eyehole and the checkerboard, in each case first marking the heights of the strings on the templet, then marking the position of the templet on the side of the box, and finally turning the templet against the side of the box with its edge on the mark we have made on the side of the box. In each case the same relations hold true, and we then make a rather naïve and childish diagram of what we have actually seen in our box, selecting for the purpose the particular case where the templet is flat against the side of the box with its two lower corners touching the corners of the checkerboard. That diagram is like figure 13.

We then simplify our diagram a little. First we draw through the point representing the eyehole a line parallel to the line that represents the bottom of the box — because this is the easiest way of getting the points that represent the eyehole and the top corner of the triangular cut in the templet at the same height in our diagram. We then leave out the

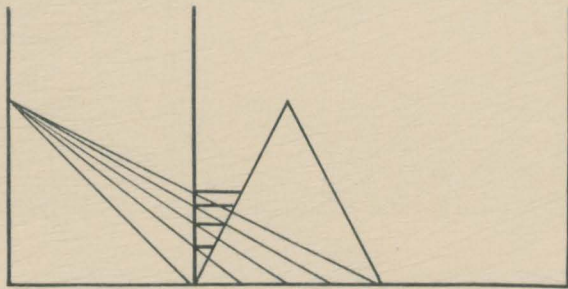


FIG. 13

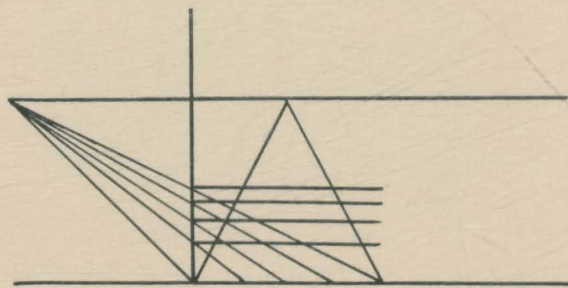


FIG. 14

ends of the box, after which we fill out the broken lines on the templet so that they run across it. These things done, our diagram looks like figure 14. Then, remembering the view from the end of the model (figs. 9, 10, and 12), we indicate the projections of the orthogonal lines on the checkerboard so that our diagram looks like figure 15. We now have a schematic drawing which we can use to throw any checkerboard into perspective. If we know the height of the observer's eye above the checkerboard (i.e. the distance BE), the length of the side of the square (i.e. the distance BC), and the distance of the observer from the square

(i.e. from D to E), we can work out our perspective on paper without having to resort to a box with strings and templet, and by making only one diagram.

After we have done this it does not take long to discover by mere inspection and no theory that a line drawn from one corner of the projected checkerboard diagonally to the far corner will pass through the corners of each of the projected squares it crosses. This diagonal line is represented by a heavy line in the diagram shown in figure 16. After we have looked at this diagram for a little it becomes

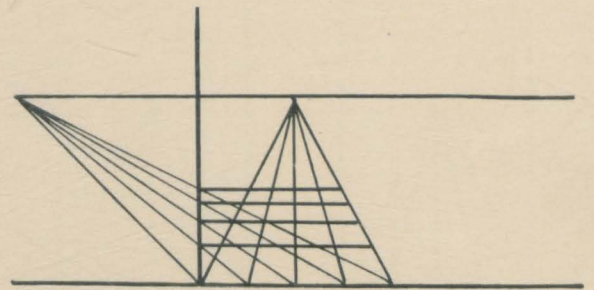


FIG. 15

obvious that all its results can be obtained in an easier manner, as shown in figure 17. From this, by way of the model, its strings, and its checkerboard, to an understanding of how to draw either irregular flat patterns or cubes and other three dimensional objects in perspective is a perfectly simple operational matter.

In many ways the most interesting thing about what we have just done is that we have done it all without any theoretical knowledge of geometry. Starting only with the knowledge that you can't see around a corner (and that therefore any line of vision is a straight line), an empty box, some strings, and a templet, we have worked out a practical method of doing perspective. The most delightful and charming thing about it is that we have done all this without knowing anything

about, or even having heard of, such things as vanishing points, or centers of vision, or horizon lines, or central lines, or cones of vision, or ground lines, or picture planes, and especially without any of the intellectual acrobatics involved in mentally revolving imaginary planes with imaginary drawings on them first about imaginary points and then about imaginary axes so that the imaginary drawings on them can come into coincidence with other imaginary drawings on other imaginary planes — all of which even the most elementary books about simple perspective ask us to

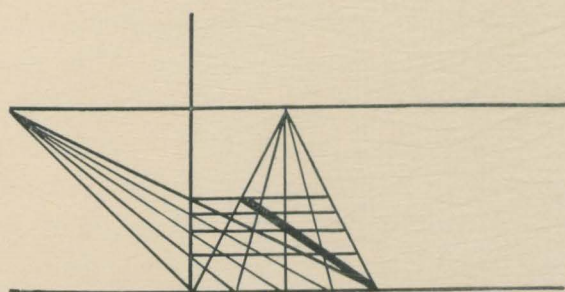


FIG. 16

do, if we want to understand perspective instead of merely following the diagrammatic prescriptions.

The last several drawings that we have made are different forms of the *costruzione legittima*, first described by Leone Battista Alberti in 1435-1436, one of which, as it was drawn by Leonardo da Vinci, is reproduced in figure 18.

III.

Now, having been through all this practical experimenting, let us make our detailed examination of Alberti's text — for unfortunately no pictures that he may have made of the several steps in his operation have come down to us, and so we must do the best we can from his text alone.

Alberti's text is free from all modern terminology and constructional ideas. Thus he

nowhere speaks of a vanishing point, a distance point, or a point of vision, an horizon line, or a ground plane or ground line. For this reason in discussing the particular passages in his text that deal specifically with his construction, I shall confine myself to his terminology.

Alberti starts his discussion with a series of theoretical considerations, in which he rationalizes the various things he has found out and invents a number of theoretical planes, lines, and points, which are to be of use to him as names for relations and positions on his per-

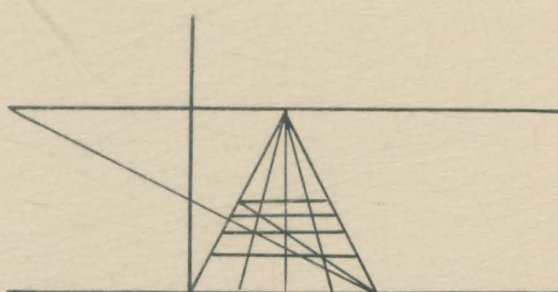


FIG. 17

spective construction as distinct from the actual relations and positions in a model. Thus he tells us that the lines of vision, which run from the eye to all points in the field of vision, form a pyramid, or, as we now call it, the cone, of vision, the apex of which is at the eye. He discovers a theoretical line that runs along the axis or center of the pyramid of vision and calls it the central line. When a plane, perpendicular to the central line, cuts across the pyramid of vision it cuts the central line at a point which Alberti calls the center point, and at which all orthogonal lines converge. The only use that Alberti makes of the center point in his construction is one for which we have used the eyehole in our model.

After finishing his theoretical introduction Alberti plunges into a description of how he makes the perspective picture on the slide of

his model — though to the discomfiture of his readers he says merely that he will show how he makes a picture (“Principio dove io debbo dissigniere,” p. 79) and leaves them to find out about the model and the slides for themselves. The object that he uses in his model and construction is a tessellated pavement, which from an operational point of view is the same as the checkerboard we have used in our model. For this reason in discussing his text I shall treat his word “pavimenti” as though it meant literally checkerboard.

Omitting his unnecessary remarks, for in the fashion of his time Alberti added lengthy classical and other allusions to the many asides that men normally lard their explanations with, let us now examine Alberti’s account of his actual method of making a perspective drawing. Without saying so he has taken his position at the end of his model, opposite the eyehole, and, looking at it, he describes the exact way in which we have arrived at the diagram shown in figure 10. He says, *I draw a rectangle, (which is to bound the picture he is going to draw on the slide of the peep show) as big as I like, which for me is like an open window through which I see whatever is to be painted,*¹⁰ that is, the checkerboard as seen in perspective. Then, explaining that the bottom of his rectangle and the nearest transverse distance in the field to be represented in his picture are proportional, he marks off the bottom line of his rectangle in as many equal portions as there are squares

(10) “Scrive uno quadrangolo di retti angoli quanto grande io voglio, el quale reputo essere una fenestra aperta per donde io miri quello che quivi sara dipinto.” — p. 79.

(11) “Et emmi questa linea medesima proportionale a quella ultima quantita, quale prima mi si traverso inanzi.” — p. 79. Alberti’s Latin version states this as follows: “Ac mihi quidem haec ipsa jacens quadranguli linea est proximiori transversae et aequedistanti in pavimento visae quantitati proportionalis.” — p. 231.

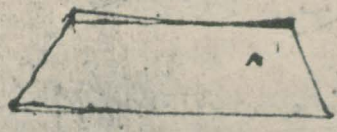
along the side of his checkerboard.¹¹

*Then, by sight, I place within this rectangle a point at the place where the central line of vision comes — because of which it is called the center point.*¹² When Alberti says that he places his center point by sight, he is not speaking quite by the book, for his center point is only the name by which he designates the indication on his drawing of the aspect of his eyehole as seen from the opposite end of his box. *Then having located the center point as said, I draw straight lines from that point to all the measured points on the bottom of the rectangle. These lines show me how transverse distances appear to change in length as they get further away to infinity.*¹³ In these last three sentences Alberti, without warning, has lapsed into a mixture of references to undescribed operation and to partially described rationalization or theory, and has done it with results that are bewildering to his reader. These sentences, to speak in terms of our model and its operation as distinct from those of Alberti’s rationalization, call for a view of the eyehole and the strings as seen from the end of the box opposite the eyehole. This is the view represented in our photograph, figure 12, and diagram, figure 9, except for the marks on the templet.

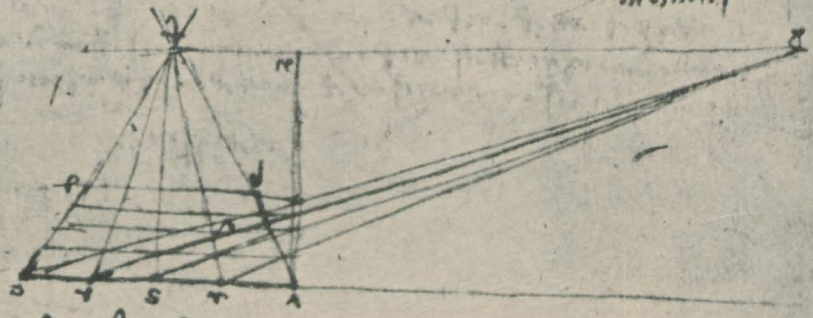
In this connection it is interesting to notice that Dürer depicts a literal eye at the point at which the orthogonal lines converge in his construction, and in his text calls that point his “nahet aug” to distinguish it from his

(12) “Poi, dentro a questo quadrangolo, dove a me paja, fermo uno punto, il quale occupi quello luogo, dove il razzo centrico ferisce; et per questo il chiamo punto centrico.” — p. 79.

(13) “Adunque posto il punto centrico come dissi, segno diritte linee da esso a ciascuna divisione, posta nella linea del quadrangolo, che giace. Quali segnate linee a me dimostrino in che modo, quasi persino in infinito, ciascuna traversa quantita segua alterandosi.” — p. 79.



Handwritten text in Leonardo's script, likely describing the geometric construction of the diagram below. The text is written in a cursive, slanted hand and contains several lines of words and symbols.



Handwritten text in Leonardo's script, continuing the description of the diagram. It includes several lines of text with some letters and symbols, providing further details about the construction.

FIG. 18. THE COSTRUZIONE LEGITTIMA AS IT WAS DRAWN BY LEONARDO

TRANSLATION BY C. Ravaisson-Mollien in *Les Manuscrits de Léonard de Vinci, Le Manuscrit A de la Bibliothèque de l'Institut*, Paris, 1881, folio 42 recto:
 “— Si tu fais un plan [carré] et que tu me le montres avec une marque ou un point qui y ait été fait au hasard, et que tu me dises seulement *s'il est fait en carré parfait ou non*, par combien de brasses a le premier côté, je saurai te dire de combien de brasses ta vue est éloignée de ce carré et à combien de brasses de distance se trouve le point fait au hasard dans ce carré, point que

nous supposons être a; tu devras faire comme il apparaît dans la démonstration ci-dessus figurée.
 “— Suis la ligne a b et la ligne d e jusqu'où elles se coupent en f; là se trouve la hauteur de l'oeil. Et si tu veux connaître la distance, tu feras la paroi [l'écran] a n, puis tu traceras la ligne c g; à son intersection avec la ligne g f est le point de distance; tire ensuite les brasses a r s t e au point f et au point g; limite ton plan, et tu verras où le point a, fait au hasard, est situé.”

“ander aug,” from which he determines the heights of his transverse lines (see p. 35 below). Leonardo, in the drawing to be seen on leaf 36 verso of his Manuscript A likewise indicates a literal eye at each of the two points. There is little doubt that in so far as Dürer understood the Italian tradition, as represented, for example, by Leonardo, he followed it in his construction, and that that tradition goes back to the operational fact that in a model the strings representing the orthogonal lines converge at the eyehole. In any event the drawing that Alberti has just described fits the facts as to both his actual

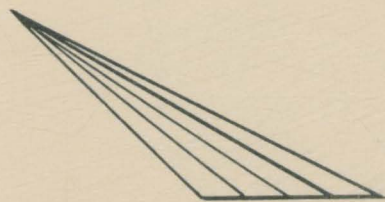


FIG. 19

eyehole and that aspect of it on his drawing which he calls his center point. On it he has indicated the way in which the orthogonal lines on his checkerboard converge as seen in perspective, the outermost of those lines providing a series of limits beyond which the transverse lines on his checkerboard as seen in perspective do not extend.

Alberti now turns aside from his explanation of his own practice to criticize an unscientific method followed by some other people, who try by an arithmetical rule of thumb to indicate the way in which a series of equidistant transverse parallel lines, lying on a plane different from that of the eye, appear to get closer and closer together as they recede from the observer.

Having finished this criticism, he returns to his own problem, saying: *But the way the transverse lines succeed one another is as fol-*

lows. Then, but without saying so, he leaves the end of his model, at which he has hitherto been stationed, and moves around to its side, and, looking at it from this different point of view, tells how he represents what he sees. *I take a little space* (“uno picciolo spatio” — doubtless a piece of paper or board on which he is going to work out the measurements that he is going to carry over onto the drawing he has started to make on his slide) *on which I draw a horizontal line, which I divide in as many equal parts as there are in the bottom line of my rectangle. Then I place a point as high above this line as the center point is above the bottom of my rectangle: and from that point I draw lines to each division in this line.*¹⁴ At this stage of his second drawing it looks like figure 19. *Then I determine the distance I want between the eye* (i.e. the point he has just placed, which represents the eyehole in his model, the precise location and height of which he knew because it was fixed in his box) *and the picture* (which is to be painted on the slide he is going to place between the eyehole and the checkerboard in his peep show), *and there I draw what the mathematicians call a perpendicular line across the other lines.*¹⁵ At this stage his second drawing looks like figure 8. He then gives a definition of a perpendicular line, and goes on: *Where this perpendicular line is cut by*

(14) “Ma nella quantita transverse come l’una seguiti l’altra cosi seguito. Prendo uno picciolo spatio nel quale scrivo una diritta linea, et questa divido in simile parte, in quale divisi la linea che giace nel quadrangolo. Poi pongo di sopra uno punto alto da questa linea, quanto nel quadrangolo posi el punto centrico alto dalla linea che giace nel quadrangolo; et da questo punto tiro linee a ciascuna divisione segnata in quella prima linea.” — p. 83.

(15) “Poi costituisco quanto io voglia distantia dall’occhio alla pictura, et ivi segno, quanto dicono i matematici, una perpendicolare linea tagliando qualunque truovi linea.” — p. 83.

the other lines gives me the order of the recession of the transverse lines. And in this way I determine the measurements of all the parallel lines and the squares upon the checkerboard as it appears in the picture on my slide.¹⁶ That is to say, the points where the bundle of converging lines cross his perpendicular give him the heights at which the transverse lines are to be drawn in the picture on the slide, and he is now able to carry them over from his working drawing to that picture and thus complete it so that it looks like figure 15.

Whether these [transverse lines] have been correctly drawn by me [in the picture on my slide] will be shown if a single straight line will form a diagonal through a number of the squares in the picture.¹⁷ His completed picture on the slide, with the diagonal marked upon it, looks like figure 20. It has been said that Alberti used this fact about the diagonal as a proof of the correctness of his drawing, and that in so doing he was mistaken because it is inherent in the geometry of his construction. This criticism, however, is not quite correct, for the critic failed to notice that Alberti had made two drawings and not one; had carried his first drawing as far as he could without certain measurements, and then to get those measurements had made his second drawing; after which, having those measurements, he had completed his first drawing. In consequence of all this he needed some way of checking up the accurate correspondence of the two drawings and sets of measurements.

(16) "Questa così perpendicolare linea, dove dall'altre sarà tagliata, così mi darà la successione di tutti le traverse quantità. Et a questo modo mi truovo descritto tutti e paralleli, cioè le braccia quadrate del pavimento nella dipintura. . . ."—p. 83.

(17) "Quali quanto sieno dirittamente descritti ad me ne sarà inditio se una medesima ritta linea continova diametro di più quadrangoli descritti alla pictura."—p. 83.

The diagonal provided the needed verification in the simplest possible way.

Just when or by whom it was discovered that the construction could be made with one drawing instead of two, by schematically turning the end elevation of the lines of vision into the same plane with the side elevation — that is, what we actually did when we turned the marked templet in our model against the side of the box — is not known. Alberti's text is not sufficiently developed for us to be certain that he knew it although there is every probability that he did. In any event that knowl-

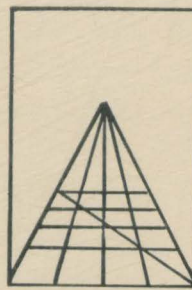


FIG. 20

edge must have followed very shortly afterwards, because Leonardo da Vinci knew all about it, including its strict relationship to actual measurement, as can be seen by examination of Ravaisson-Mollien's facsimile and translation of leaf 42 recto of Leonardo's Manuscript A (fig. 18).

Alberti has now arrived at the end of his technical description of his perspective construction, but he goes on to say: *This being done, I draw a straight line across the picture on my slide from side to side, parallel to its bottom line and passing through the center point, and thus divide my picture. This line serves as a limit above which nothing in the picture can extend that is not higher than the eye of the observer. And because this line passes through the center point I call it the central line. From this it follows that the fig-*

ures painted on the furthest squares of the picture are smaller than the others; as Nature demonstrates to us.¹⁸ This statement is interesting as showing how near Alberti came to discovering the idea of the horizon line, which in effect he had when he had drawn his line across his diagram through its center point.

It is worth while to go back, at this place, and consider a little more fully Alberti's phrase about determining the distance between the eye and the picture (see p. 15 above) — for it is this phrase which, although one of the greatest stumbling blocks to an understanding of his text, contains the key to the operation described in it. When he wrote his description he was thinking in large part in terms of his model and the things that he actually did with it. Thus the position of the slide (which for many purposes is identical with the templet in our model) was the only variable he mentioned, for the eyehole and the checkerboard were built into his box so that in his experiments their locations and measurements were constant — though no one knew better than he did that if the eyehole were made higher or lower or moved to one side or the other, or if the checkerboard were slid along the bottom of his box in either direction, then the size and height of the checkerboard as represented on his slide (or, in terms of our model, the pattern of the points where the strings pass through the templet) would be different, and the lines of vision (or

(18) "Fatto questo, io descrivo nel quadrangolo della pictura ad traverso una dritta linea dalle inferiore equedistante, quale dal uno lato all' altro passando su pel centrico punto divida il quadrangolo. Questa linea a me tiene uno termine, quale niuna veduta quantità non più alta che l'occhio che vede, più sopra giudicare. Et questa perchè passa pel punto centrico dicesi linea centrica. Di qui interviene che li huomini dipinti, posti nell ultimo braccio quadro della pintura sono minori che gli altri; qual cosa così essere la natura medesima ad noi dimostra." — p. 83.

strings) would make different angles with each other both at the eyehole and at the floor of the box. Every man who actually works with a tool or device such as the model I have described knows this kind of thing simply from working with it, and needs no explanation of it, verbal or otherwise. The principal reason for his usual failure to call attention to it or to explain it is its complete operational obviousness to him.

One of the greatest difficulties that men have in understanding explanations arises from the fact that the explainer tacitly thinks in terms of one particular tool or operation with its inherent, disguised factors, and the explainee tacitly thinks in terms of a different tool or operation with its different inherent, disguised factors. It was this even more than Alberti's obscurity of expression that for a long time prevented his discovery from becoming known among the artists. Where Alberti thought in terms of a picture on a movable slide in a model, the ordinary artist was trying to learn how to make a picture that could hang or be painted on the wall of a room and that would appear to be adequately in perspective no matter what its distance was from the eye of its beholder. This difference in problem can be phrased by saying that as between the constants and variables of Alberti and the ordinary painter, the only thing in common was the constancy of the size of the checkerboard. Because of this it took the painters in general a long time to discover that Alberti's construction could be adapted to their purpose by simply anchoring the picture plane to the checkerboard in such a way that the bottom line of their picture coincided with the near transverse edge of the checkerboard, and that once this was done the only measurements they required were those representing (1) the dimensions of the checker-

board, (2) the distance between the observer and the near side of the checkerboard, and (3) the height of his eye above it, that is, three measurements which were so fixed and obvious in Alberti's particular operation that he forgot even to mention two of them. This anchored position of the picture plane is that actually illustrated in the photographs of our model.

IV.

On turning from Alberti to Viator, we find another construction, which, as pointed out above, has precisely the same results as that of Alberti. Thanks to The Pierpont Morgan Library it is possible to give in an appendix a facsimile of Viator's French text, which seems not to have been reprinted in modern times. Because of the light that they throw upon Dürer Viator's schematic diagrams are here reproduced from the Nuremberg piracy.

As Viator acknowledges in his last paragraph, his diagrams are of much greater importance than his text, and for their understanding require not so much words as a sense for the business in hand. His text, however, is interesting, because it contains in all probability the first printed references to the ground plane, the horizon line, and the "tiers points" from which his system got its name of three-point perspective. The most important single statements in his text are those in which he says that his center point and his two distance points are located on a line at the level of the eye, and that his two distance points are "equedistans du suiect: plus prochains en presente, et plus esloignez en distant veue." As otherwise his words add very little to the analysis we have to make, we shall, to save time, confine ourselves to his diagrams.

Just as Alberti, in his text, starts off with a series of rationalizations of his actual operation and then proceeds to describe his opera-

tions in terms of his rationalizations, so Viator follows the same procedure in his drawings. He begins with two theoretical explanatory constructions and ends with the three simplified diagrams of his actual working construction — thereby, like the writers both of detective stories and of studies like this, reversing his actual order of work and telling the story of his discoveries backwards.

The typical working diagram for Viator's construction is his third woodcut (fig. 26). We are familiar with this, as still further simplified, in figure 21 (see also p. 15). It has been

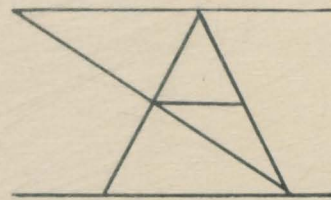


FIG. 21

suggested that this diagram, or construction, possibly represents a tradition or method that had been in use among the French masons of the late Middle Ages and early Renaissance, but, so far as I have been able to discover, no specific reasons for this conjecture have been adduced. It may well be based simply upon Viator's textual use of several terms that formed part of the French masons' vocabulary — a species of argument that if applied to the present study would result in the charming discovery that it represents a tradition among American carpenters.

Often as it has been pointed out that Alberti's and Viator's constructions give identical final results, it seems to have been considered that in its simplified form (as in fig. 21) Viator's construction is much more abstract than Alberti's, and that therefore it represents a much more considerable effort of geometrical imagination and knowledge than Alberti's does. I have nowhere met any sug-

gestion as to how it might have had its origin in an operation, as distinct from the geometer's theorems and analysis. It would seem, however, that it may well have evolved from a simple mechanical operation. If we again revert to the use of a model, this will be apparent — as will also be the fact, which seems not to have been mentioned hitherto, that in spite of its apparent unlikeness to the construction of Alberti the construction of Viator represents the same series of operations with but one very slight modification.

If we may predicate that Viator, a much

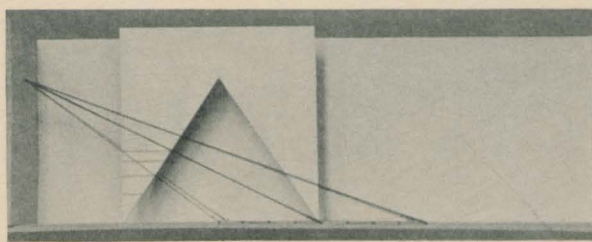


FIG. 22

traveled and intelligent man, hearing or learning that Alberti, as the *Vita anonyma* says (see p. 16 above), "showed these things through a tiny opening that was made in a little closed box," himself began to experiment, much as we have done, there will be no difficulty in working out a way by which he might have discovered his particular construction. As we have seen, Alberti, by working with a model such as that which we have used, could well have discovered the *costruzione legittima* by merely swinging his templet against the side of his box in such a position that its two lower corners came in contact with the corners of the checkerboard. In doing this the templet was swung as though it were a door hinged to the side of the box in such a way that its two lower points could coincide either with the two corners of the checkerboard nearest the eyehole or with the

two corners of the checkerboard next to one side of the box.

Viator, using a similar apparatus, could have achieved his particular construction by revolving his templet on its apex as a pivot rather than by swinging it around on its perpendicular edge as a hinge. As revolved in this manner, and not swung, its appearance and relationships are shown in our photograph, figure 22, in which, to make the matter more obvious, several of the unimportant strings have been omitted. Comparison of this photograph with the preceding simplified schematic diagram (fig. 21) shows that this abstract geometrical construction is, like the *costruzione legittima* of Alberti, very little more than a childishly naïve picture of something that can actually be seen in a simple model.

By revolving his templet about its apex, Viator kept that apex in its original position, directly above the center of the near edge of the checkerboard, while Alberti, by swinging his templet doorwise about a perpendicular edge, moved the apex of the templet away from its original position to one directly above the center of an orthogonal edge of the checkerboard, and thus left only the hinge edge of the templet in its original position perpendicular to the near edge of the checkerboard. This shift in the position of the apex of the templet explains why it is that although the crucial distance between the near edge of the checkerboard and the observer is measured off, in Viator's scheme, from the apex of the templet, or center point, in the *costruzione legittima* it is measured off from the perpendicular — for each of them, apex and perpendicular, in its particular construction, stands for the near edge of the checkerboard. It was because Dürer did not clearly understand this basic operational difference between the two

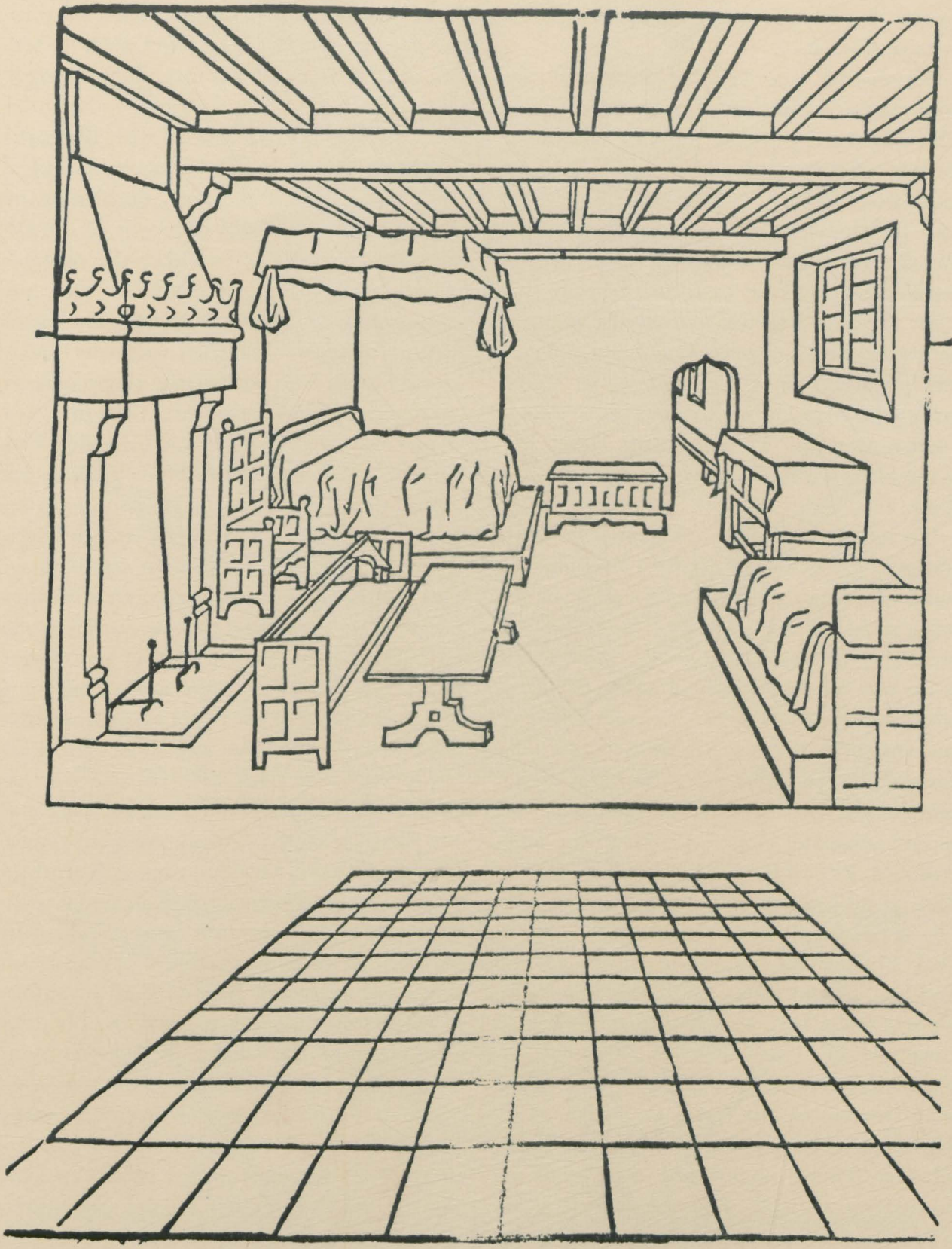


FIG. 23. WOODCUT OF VIATOR'S LIVING ROOM

systems that he came to grief in his own perspective scheme.

Whether or not Viator's solution of the problem was worked out first by Viator himself or by some artist, builder, or craftsman, there is, of course, no way of knowing. It is quite possible that Alberti, while working with his model, may have discovered it, and also that he discarded it for his other discovery of the *costruzione legittima*, because this latter, by enabling him to draw the pictures he needed in the correct sizes and at the correct heights on the several slides of his model, made it possible for him to make his academic "demonstrations" of his theory about the pyramid or cone of vision and its cross sections.

To return to Viator's diagrammatic illustrations: Just as he revolved his templet (instead of swinging it against the side of his box), so in his first diagram (fig. 24), intent on theory or rationalization, he represented at the bottom of a pyramid of vision, the circle which was the trace of the two lower points of his templet as it revolved about its axis (the apex of the pyramid being the apex of his templet and the perpendicular the axis). His second diagram (fig. 25) shows the same pyramid, but with its bottom tilted back as seen in perspective. His third diagram (fig. 26) is a schematic simplification of his second. His remaining diagrams (figs. 27, 28, and 29) are no more than various applications of the construction arrived at in his third diagram. His third diagram, however, was the all-important one — and it is only a slight amplification of our figure 21, which, as we have seen, is little more than a naïve picture of what Viator might have seen when he looked into a model. Thus the order in which Viator undoubtedly worked out his diagrams was not that in which he printed them in his

book, but something much more like this: first our figure 21, next his third, then his second, and last of all his abstract and theoretical first diagram.

To anyone who is familiar with fifteenth-century and early sixteenth-century pictures, and especially with the book illustration of that time, the first sight of a copy of the *De artificiali perspectiva*, in either the original Toul edition of 1505 or the Nuremberg piracy, comes with a sort of a shock. The many pictures of known buildings with which Viator exemplified his perspective method are so clear, so reasonable, so just, that they are wholly out of tune with anything that had been done before them or that was done for a long time afterwards. Such a sudden step across the centuries into a completely modern system of pictorial organization and point of view can hardly have happened on any other occasion. Dürer's Saint Jerome in His Study (B. 60) of 1514 is a portrayal of an interior that is famous for its mastery of perspective, but as compared with Viator's representation of his own living room, published nine years earlier at Toul and five years earlier at Nuremberg, it gives no feeling of space and no visual comfort. Where we can believe Viator's picture with our eyes as a truthful report of something that had an actual existence, our attitude towards Dürer's engraving is that with which we listen to a charming but obviously impossible fairy tale, in which there is no reasonability and no hard fact. Our reproduction of the woodcut of Viator's living room (fig. 23) is made not from the Toul original but from Georg Glockenton's coarse copy that was published at Nuremberg. It is presumably the same as that which appeared in the original edition of the Nuremberg piracy five years before Dürer in that same town gave his Saint Jerome to the world.

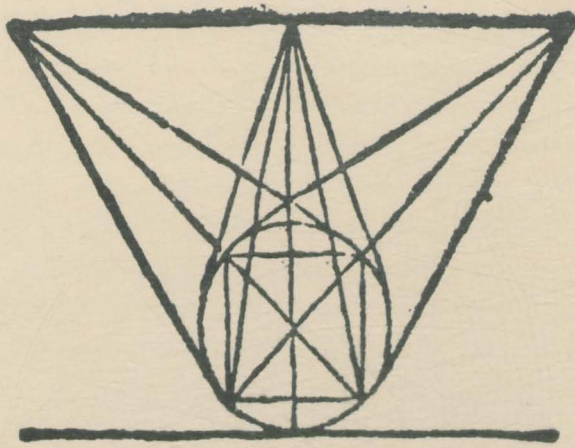


FIG. 24. VIATOR'S FIRST DIAGRAM

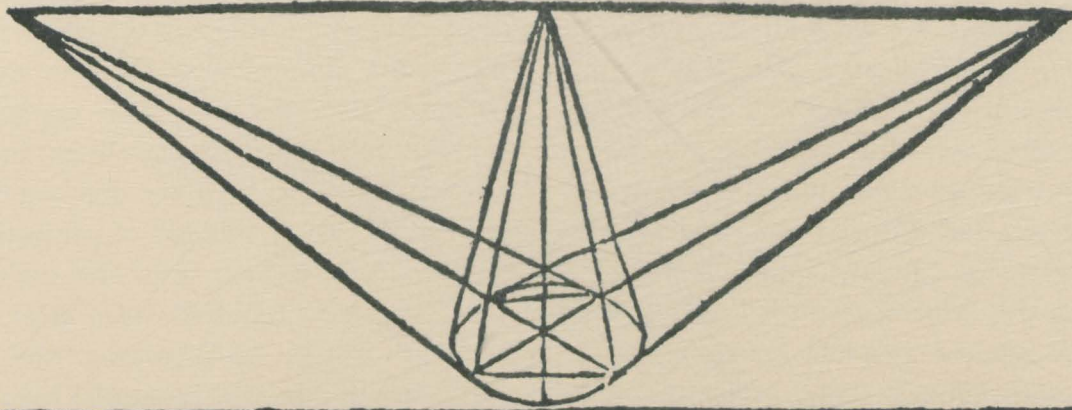


FIG. 25. VIATOR'S SECOND DIAGRAM

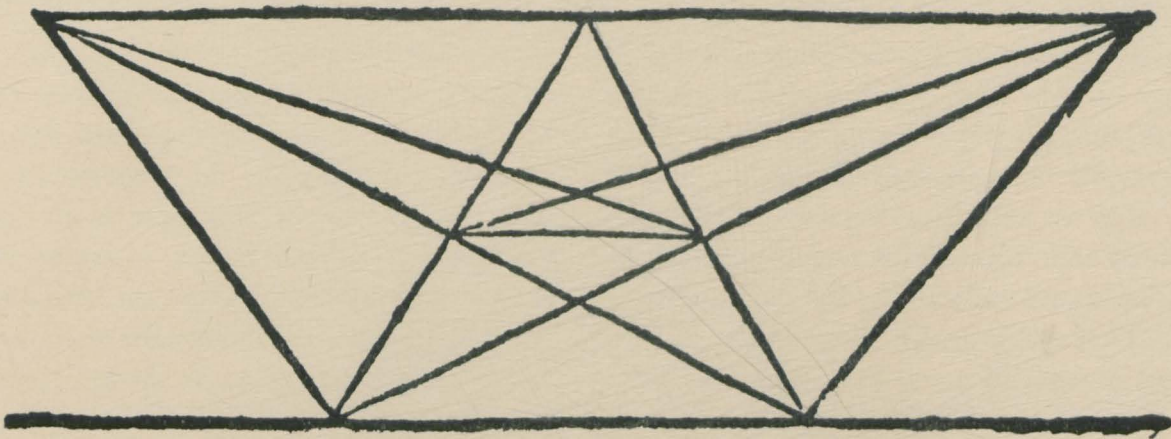


FIG. 26. VIATOR'S THIRD DIAGRAM

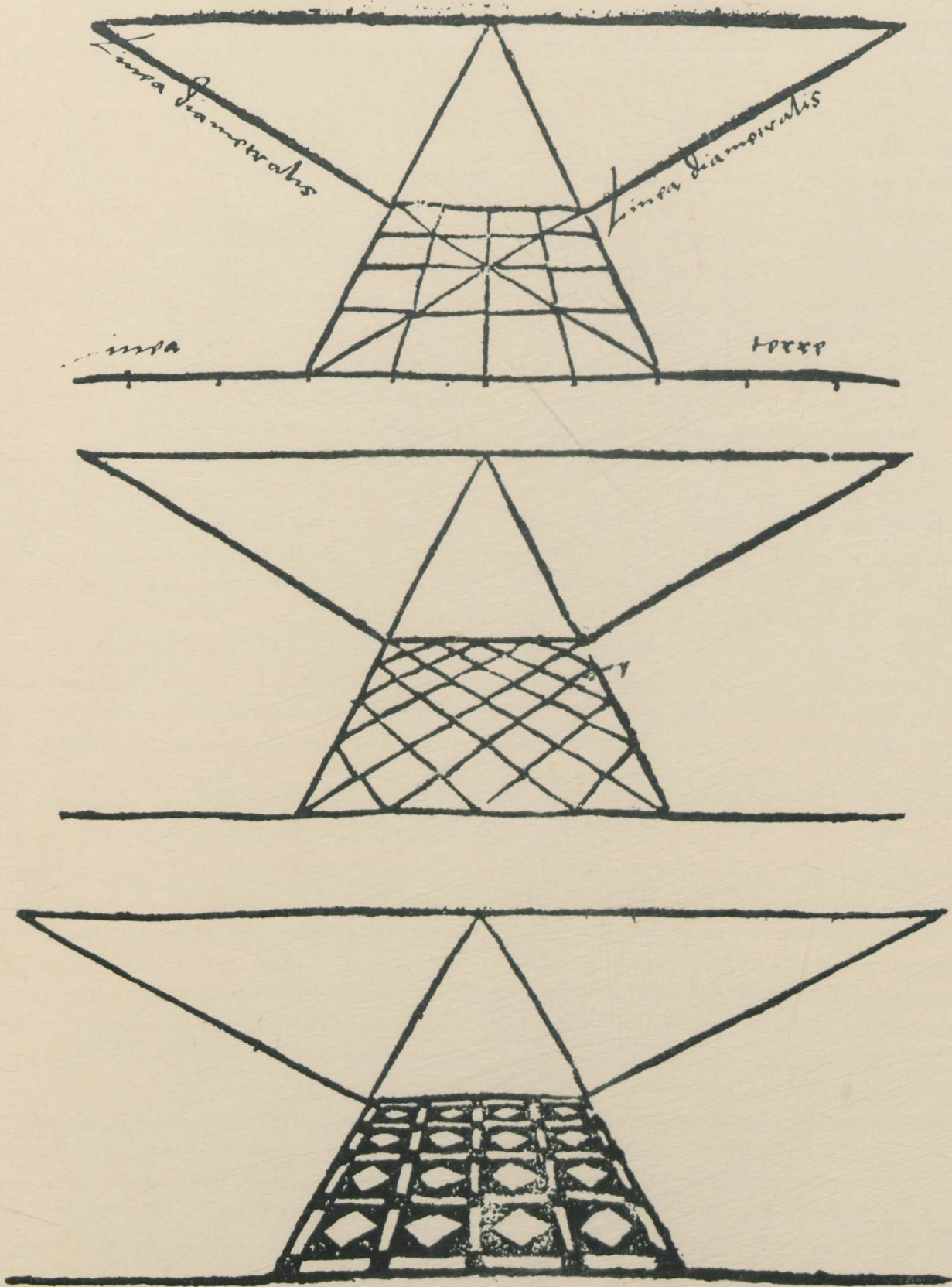
V.

It has been said that Alberti's great discovery was that the picture plane was a perpendicular cross section of the cone of vision, but there is reason to believe that possibly it was something more and other than that. Until Alberti's time the problem seemingly had been confined to a simple two-term relation between the beholder and the single object, in which the beholder saw only the object and no one saw the beholder. So long as the problem was confined to a particular object as seen through the beholder's eyes it remained strictly insoluble, because the beholder, like John who had hold of the bear's tail, could not stand off and survey the situation. Alberti's great stroke of genius lay in his practical realization that the problem was not to be solved by thinking only about the bear as seen through John's eyes, but that what Henry and Thomas saw from the side lines had to be brought into consideration. In doing this Alberti discarded an insoluble two-term relation and took on a series of relations with enough terms to permit of its solution. In other words Alberti discovered that, pictorially at least, form and position were functions of each other, and thus were relative and not constant, and also that there could be no statement of position in three-dimensional space in anything short of a three- or four-term relation. This again is merely another way of saying that by adopting a particular geometrical convention Alberti was able to substitute something that was rational and objective for something that was irrational and subjective. By getting Henry and Thomas to make diagrammatic statements of what they saw from their respective positions on the side lines, Alberti came into possession of the diagrams reproduced in figures 8 - 15. He told how he correlated these

various statements in that portion of his text analyzed on pages 22 - 25 of this paper. One of the ways in which it may have been discovered that the different statements could all be made in one single diagram has been explained on page 19 of this paper. Later on Viator achieved identical final results by proceeding in a slightly different way, as has been shown on page 28 of this paper.

The solutions of the perspective problem that are associated with the names of Alberti and Viator were based upon the simplest kind of practical ingenuity, and in some respects were little more than clever carpenter's work. The two solutions were full of implicit mathematical relationships, but the men who used them were content with them as easy contrivances that worked. The mathematical analysis of the perspective problem, and of the special variety of geometry that was implicit in Alberti's novel method of projection and section, seems to have been first undertaken, just about two hundred years after Alberti wrote his treatise, by Desargues, who utilized an assumption by which parallel lines concur at a point at infinity (see p. 11 above).

As a result of the work of Desargues and his nineteenth-century followers, there has been developed out of the Albertian perspective construction what is possibly the most generalized discipline of geometrical thought. The ordinary pictorial perspective diagram of today, while retaining the outward form of Viator's construction, is customarily explained in terms of this branch of higher mathematics - that is to say, in terms which are so rarely understood that the actual practice of perspective is merely the routine of a memorized prescription. In modern perspective, the "vanishing point" is the name for the "projection" on the "picture plane" of the "point at infinity" at which the group of "or-



FIGS. 27-29. VIATOR'S FOURTH, FIFTH, AND SIXTH DIAGRAMS

thogonal lines" "converges." The modern vanishing point is thus a matter of highly technical definition. It is not only completely foreign to the renaissance idea of the center point, which, as we have seen, was merely the operational name for a particular aspect of the observer's eye, but it is based upon an assumption completely foreign to the basic assumption of classical geometry that parallel lines could never meet no matter how far extended. Such "questions," therefore, as: "Did the ancients know the vanishing point?" are strictly comparable to such other "questions" as: "Did the ancients know the square root of minus one?" Both the vanishing point and the square root of minus one exist only by virtue of assumptions and definitions first made long after the ancients had ceased to exist. Thus when learned classical scholars translate "obscurum acumen coni" as "the vanishing point of a cone," they not only make nonsense of Lucretius's sensible words but impute to him their own misunderstanding of definitions and technical terms with which he could not possibly have been acquainted.

VI.

Dürer's prominence as an artist, the wide pervasion of his books and prints, and especially the fame and popularity of his theoretical writings, justify us in looking at his actual accomplishment in perspective as closely as possible. Dürer certainly was familiar with the linear appearance of Alberti's diagram, and possibly even with Alberti's account of his method. In spite of what has been accepted as fact by practically all the commentators, I personally cannot believe that Dürer was unacquainted with Viator's diagrams, which were published in the Nuremberg piracy of Viator's book in 1509. That piracy, we must not forget, was the most important book on

the subject of Dürer's predilection printed in Dürer's home town prior to the publication of his own book in 1525. That he should not have seen and known it is utterly incomprehensible except at the cost of his reputation for being really interested in its subject. A set of later impressions of the very rare woodcuts for the Nuremberg piracy is in the Print Room of the Metropolitan Museum, but the text that should accompany them is unfortunately lacking. These woodcuts are coarse but schematically perfect copies of those in the original French edition of 1505. As we have just seen, Viator's woodcuts contain the gist of his matter, and so any possible stupidities in the text that accompanied the pirated copies of them can hardly constitute an excuse for any failure to understand them on the part of a great man with a great reputation as a scientific thinker.

The difficulty that Dürer had with the perspective systems of his two predecessors was doubtless due to his failure to understand the operational bases from which they had been evolved. In all likelihood it was the operational basis of Alberti's system that Dürer wanted so much to learn about during his second Italian trip — and that he never did learn about. When we come to look at the celebrated diagram 59 in Dürer's *Unterweysung der Messung* and his attempted explanation of its construction, we shall see that he came to grief precisely in those places where his predecessors were not explicit about what they were doing.

Where Alberti had a model containing a checkerboard, Dürer had a cube standing on a square surface, and his problem was to represent in perspective the shadow cast by the cube on the square surface. Before he could throw either his cube or its shadow into perspective he felt that he had to throw the square

surface itself into perspective. For practical purposes this last problem was identical with the problems of both Alberti and Viator, but it was so differently expressed and was undertaken with such a different purpose that the identity was perhaps not as obvious as it might have been.

Dürer had two methods of arriving at his picture of the cast shadow, one longer and more complicated, the other shorter and easier. This easier one was his celebrated "nahere weg," as he called it, and the one on which his fame as a master of the theory of perspective is based. Dürer's text of this, in so far as it deals with the projection of the square surface on which his cube rested, is given at the bottom of this page¹⁹ in a facsimile from the 1538 edition of his *Unterweysung der Messung*. His diagram 59 is reproduced as figure 30.

In his description of his nahere weg Dürer starts with a transverse line drawn across his paper. A little above the left end of this and

parallel to it he draws another line $g f$ of the same length as the edge of the square surface to be projected in perspective. Above the line $g f$ he indicates a near point of sight ("ein nahet aug") with an eye, placing it at the same distance above that line that it occupied in one of the diagrams for his more complicated construction (none of which is here reproduced) — which was anywhere that he wanted to put it. From this near point of sight he draws lines to the points g and f . In this way he has indicated the near side of the square surface as seen in perspective and the directions of the two orthogonal sides. His remaining problem is where and how to draw his fourth side. Off to the right, and at the same height as the near point of sight above his transverse line, he indicates a second point of sight ("ein ander aug") by drawing another eye, but this time placing it in the head of a man who stands with his feet on the transverse line. The distance at which this second point of sight is set off is the same as in Dürer's

(19) **S**ernach wil ich durch'eyn' anderen vnd neherm weg /gleich das vorbeschriben ding abgestolen/in das gemel leren pringen. Durch ein solchen weg.

Leg vber zwerch ein lini in der leng der forigen .e. f. g. h. des forderen außgezognen grundes /die da an stat einer gefierten ebenen ist / vnd setz ein nahet aug auf der seiten ob der lini .f. wie dann das for auf dem puncten des aug der kreuz linien stet bey dem for beschriben ding.

So das gemacht ist als dann zeuch auß disem aug zwo gerad lini an bede ort der nider gelegten lini .e. f. g. h. die machen vnden zwey eck /vnd der stierung sind drey seiten gemacht die vierecket abgestolen sollen sein. Nun must du die hinder seiten wissen zu machen /wie hoch sie vbersich steygt / das sind also. Setz ein ander aug auf die seitten in der weyte wie das bey dem for beschribnen grund stet aber gleych in der höch wie das neher aug. auß disem aug zeuch zwo gerad linien an bede ort der fürgelegten linien. Darnach reiß ein aufrechte lini .aa. bb. die daß forder eck an rürt /wo dann dise außrecht lini abschneydt / die lang streym lini die auß dem weyterem aug in den spizigen winkel zogen ist in den puncten setz .cc. Auß disem puncten .cc. zeuch ein zwerch par lini durch die zwo streym linien die da auß dem nahetten aug auf die zwey vnderen ort der zwerchlini der stierung zogen sind .wo dvnne die streym linien durch schneiden werden /da machen sie zwey eck /also ist dise gefierte /ebene rechte abgestolen /gleych wie die for gemacht /darumb bezeichnen auch jr vier eck mit den vier buchstaben .e. f. g. h. wie hie foren im ersten gethan ist. Disz hab ich hienach also außgerissen /ec ich weyter gee.

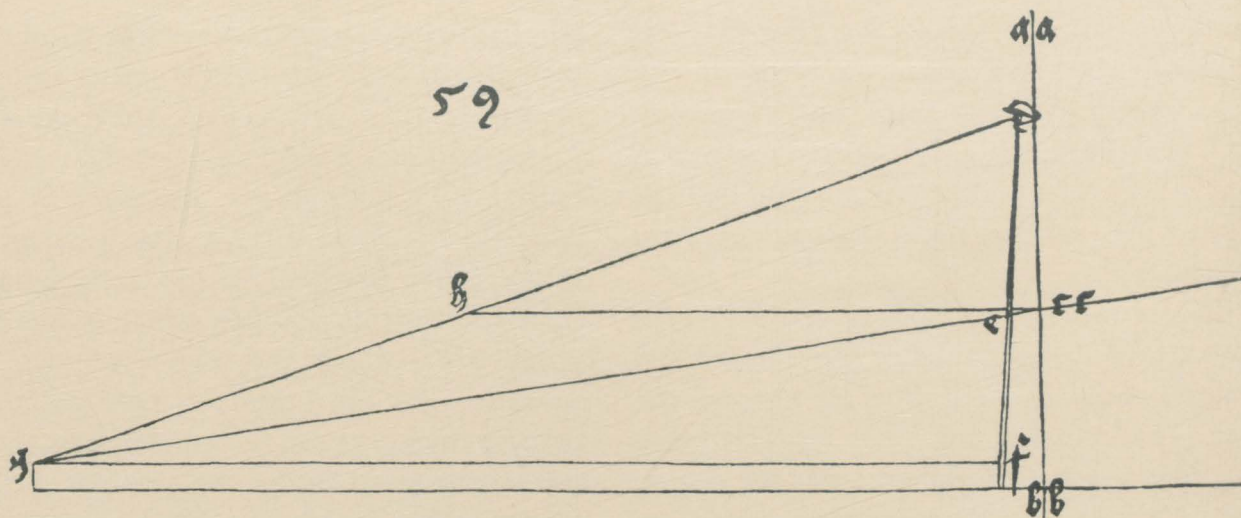


FIG. 30. DÜRER'S

more complicated construction. In his description of his more complicated construction Dürer says that he puts the second point of sight as far from the square surface as he likes ("so weyt . . . als ich will"). The distance of this second point of sight is thus set with absolute arbitrariness. Having indicated his second point of sight he says that he draws lines from it to the points *g* and *f*. (He actually only draws the line to the point *g*.) This done he says that he draws a perpendicular line *aa bb*, which touches the near corner ("die das forder eck an rurt"), that is, at *f*, but which in his diagram actually stands off to the right of *f*. Finally from the point *cc*, where this perpendicular cuts the line between the second point of sight and the point *g*, he draws a transverse line parallel to the line *gf*. At the points where this line cuts the lines from the near point of sight to *g* and *f* he puts the letters *h* and *e*. The line *he* is the missing fourth side of his projected square.

In proceeding in this way Dürer sets off his second point of sight "so weyt als ich will," but without saying from what point its distance was measured off, whether from the near point of sight or the point *f*. It is important

to notice that he sets off this second point of sight before he erects any perpendicular. This would have been in accordance with Alberti's method of procedure within the confines of his box with its fixed, constant positions of eyehole and checkerboard, its shifting, variable position of slide or templet, and the variable size and position of the picture on the slide. But it was wrong when working on a piece of paper, as Dürer did, with shifting, variable positions of eye and checkerboard, and constant position of the picture plane at the near edge of the checkerboard. If Dürer measured the second point of sight off from the near point of sight, then his perpendicular, far from being of any help to him, was only a very disturbing element of confusion and error, because when the construction is based on the distance between the two points of sight, the true geometrical diagram is that shown in figure 21, which is the construction of Viator and contains no perpendicular. If he measured his distance off from the point *f*, he would have had to use one or the other of two ways of carrying that distance up to the central line (which Dürer actually omits) from the ground line, on which the point *f*

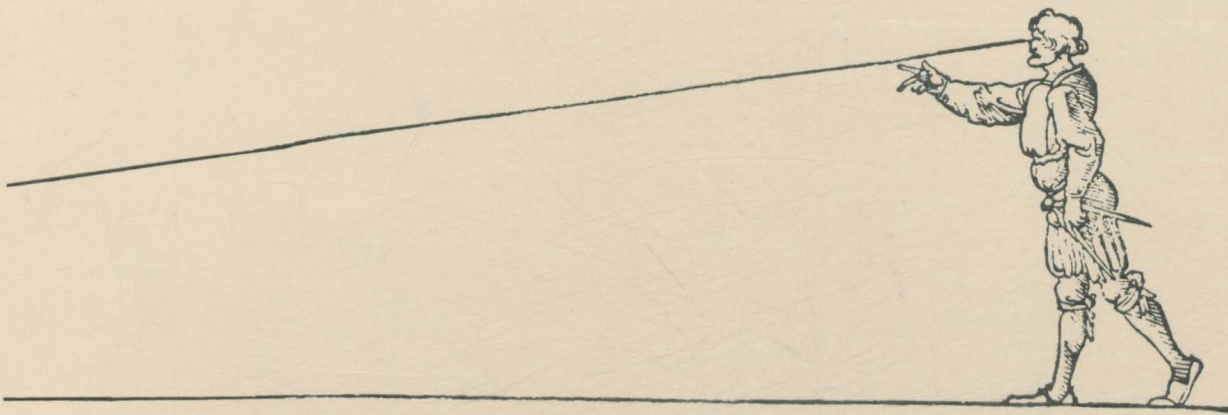


DIAGRAM 59

lies. He could have drawn a perpendicular through the point f and taken his measurement from the point at which the perpendicular cut the central line, as in figure 31. But

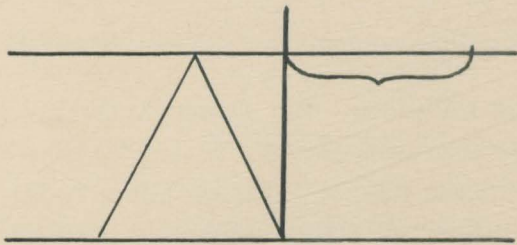


FIG. 31

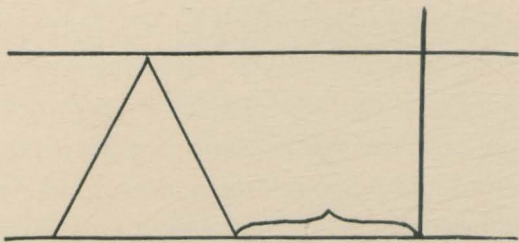


FIG. 32

this he did not do, because he located his second point of sight before he erected his one perpendicular. The only other way was to measure his distance from the point f along the ground line and then transfer it to the central line by erecting another perpendicular

that cut the central line at the proper place, as in figure 32. But this form of construction requires two perpendiculars, a first one to determine the position of the second point of sight, and then another through the point f , to be used in determining the height of the fourth side of the projected square. As Dürer's description and his diagram contain only one perpendicular it is obvious that his construction was not made in this way.

Further than this it is interesting to note that Dürer did not in fact draw his one perpendicular through the point f but off to one side of it (in a position which logically should have required the drawing of the line, omitted by Dürer in his sketch, from the second point of sight to the point f); and that he put the feet of his observer on the lower transverse line (the ground line) instead of on the plane of his square surface, and thereby introduced two unrelated ground lines into his diagram. As part of this last error he made the lower left-hand corner of his construction like figure 34, when, if he had really wanted to put his two points of sight at the same height above his original ground line, it should have been like figure 35. Each of these several things in-

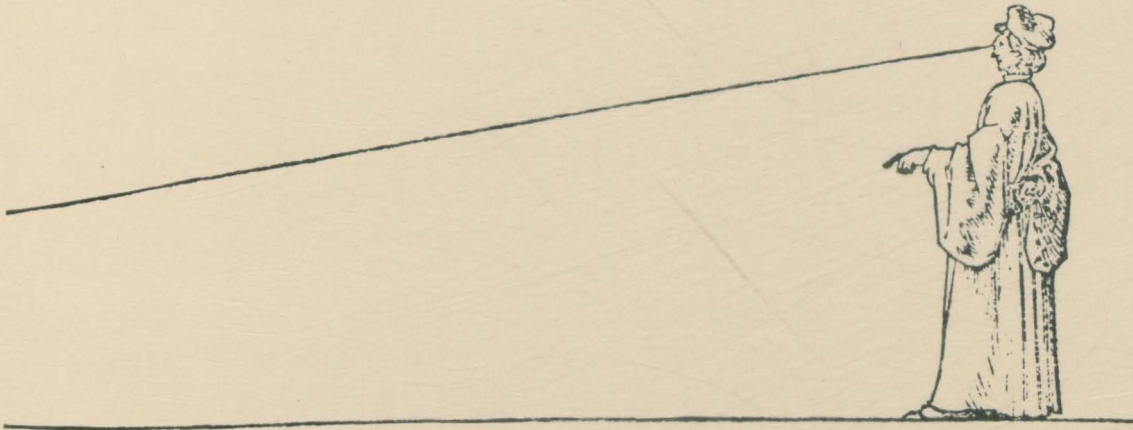


DIAGRAM 61

tween his near point of sight and his perpendicular, and thus succeeded in getting a "photographic wide angle" distortion into his perspective renderings.

If we remember how the minds of schoolboys and other bewildered people work, we get another hint about Dürer's possible knowledge of Viator as well as of Alberti in the fact that in all the constructions in his *Unterweysung* he places his near point of sight very close to his perpendicular. Had he known only Alberti, or only Viator, he would not have needed to do this and could have put his near point of sight out in the middle of his picture where it would have been most useful to him. But, being acquainted with the systems of both his predecessors and understanding neither, he was able to reconcile them only by the expedient of placing his near point of sight as nearly as possible in the one

theoretical position where there was little or no practical difference between them. It happens (see p. 14 above and fig. 36) that when the perpendicular and one of the lines from the near point of sight to the base line coin-

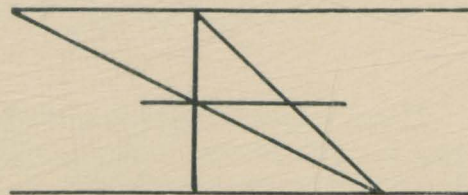


FIG. 36

cide there is no ostensible difference between the constructions of Alberti and Viator. Thus by drawing the constructions in his book with the near point of sight very close to the perpendicular, Dürer played safe in the one general position where the error caused by his confusion of the two systems was the least

troublesome. He used this position in many of his woodcuts, and notably in his engraving of Saint Jerome in His Study (B. 60). While this enabled him to satisfy his pedantic yearning for theoretical correctness, it forced him to place his vanishing point away off towards the edge of his picture in a position, which, when emphasized by his "wide angle" distortion, has the effect of making any picture so constructed appear, in a subtly disturbing way, as though it were only half a picture. This peculiarity of construction is so familiar in Dürer's work that it may almost be thought of as one of his distinguishing characteristics as an artist.

We have examined with some care Dürer's diagram 59, in which he takes the first step towards the solution of his problem of how to throw into perspective the shadow cast by a cube resting on a square surface. Let us now pass on to his diagram 61 (our fig. 33), in which he gives his complete answer to the whole problem. This diagram is a cumulative one, purporting to contain all the constructional steps taken from the beginning of the problem to the end of it. In so far as the perspective of the square surface is concerned, therefore, it should be exactly like his diagram 59. But this is not the case, for here

Dürer clearly and definitely throws his square surface into perspective by Viator's method, finding the height of the fourth side of his square by the point where the line from his near eye to the point *f* intersects the line from his second eye to the point *g*. The Albertian perpendicular is there, to be sure, but it is sheer surplusage, serving absolutely no constructional purpose whatever. That useless perpendicular is the fitting sign and proof of Dürer's quality as a thinker and geometrician. It is learned, it complicates matters, and, except in so far as it shows that Dürer did not understand what he was about, it is devoid of meaning.

Dürer's next diagram is reproduced as our figure 37. Having been added to the second edition of his *Unterweysung*, it does not bear one of the running numbers that were cut on the blocks for the first edition. The text of Dürer's explanation of it is given in the facsimile below.²⁰ In this he purports to tell us how to throw a point into perspective — but, characteristically, he makes a crucial error. In his diagram the picture plane, as determined by the three points, *o*, *a*, and *b*, intersects the ground plane along the axis *a b*, that is, at the side of the square furthest from the observer. To phrase this in terms of our model he has

(20)

Wenn du in einem abgestolnen plarten ein puncten finden wilt/der dir in eyner rechten fierung fürgeben wirdet/dem must also than/Sez ein rechte fierung.a.b.c.d.also das.a.b.oben zwerchs sey darnach sez die abgestolne fierung.a.b.g.f.oben an die ander/vñ das .aug darzu sey.o. dann sez in die rechte fierung einen puncten .e.wo du hin wilt /darnach reiß in die fierung ein ortstrich.a.c.also reiß auch ein gleichmessigen diameter.b.f.in die abgestolne fierung/darnach reiß aus dem gegebenen puncten.c. eyn parallel lini gegen oder mit der seitten der fierung bis an die zwerchen .a.b.dahin sez ein.h.von dann reiß durch die abgestolne fierung ein gerade lini gegen dem aug.o.bis an die zwerchen.f.g.da sez ein.m.darnach reiß in der rechten fierung/ ein gerade paralellini aus dem puncten .e.bis an den Diameter .a.c. da hin sez .ein.i. von dann far mit einer aufrechten parlimi bis an die zwerchen.a b.da hin sez ein.k.von dann reiß zu der abgestolnen fierung gerad gegen dem aug.o.bis an den ortstrich.f.b.da hin sez ein.l.von dann far parallel ob .a.b.zwerchs an die lini.h.m.da hin sez ein.n.Diß ist der gefunden punctt.in der abgestolnen fierung. vnd stet gleich messig in seinem teyl.wie der punctt.e. in der vnderen rechten fierung/Disse figur hab ich nachfolget also auf gerissen.

placed his templet at the far instead of at the near side of the checkerboard. In this position of the templet it is impossible by its means to get a perspective image of the square and a point on it except on the supposition that the templet is not a templet but a mirror, in which case the projected image would of necessity be a mirror image, that is, a reversed image. Dürer, however, reverses one of his two diagonal co-ordinates, so that they no longer intersect as they should on the axis of his two planes. By so doing he gets the perspective image n of his point e , not at the top of the projected square $abgf$, as it should be in a mirror image, but at its bottom, where it could not possibly be located if his ground and picture planes intersect along the line ab . Because of the familiarity of various modern conventions of convenience on the drawing board this is rarely noticed. For Dürer suddenly and knowingly to adopt a factually erroneous convention merely because of its convenience on the drawing board would be completely out of character both for him and for his time. Not only does Dürer nowhere indicate that he is actually resorting to such a convention in the making of this diagram, but such a convention would be in direct conflict with his correct statement in his description of his more complicated method (on leaf ii verso of the 1538 edition of the *Unterweysung*) that his picture plane is a transparent plane, or flat field, which cuts across the lines of sight ("ein durchsichtiger planus oder eben feld der all streymlinien durchschneidet").

The effect of Dürer's misunderstanding of the meaning and use of the perpendicular and

(21) "Sara bene posto questo punto, alto dalla linea che sotto giace nel quadrangolo non più, che sia l'altezza del huomo quale ivi io abbia a dipigniere; pero che cosi et chi vede et le dipinte cose vedute, pajono medesimo in su uno piano." Alberti, Janitschek's edition, p. 79.

the two points of sight was frequently heightened by his failure to obey Alberti's behest to place the center point (or, as Dürer called it, his near eye) at the height of the heads of the figures in the immediate foreground.²¹ Far from doing this, he most frequently placed it well above them. This trick or device of a high center point, had Dürer carried it out in

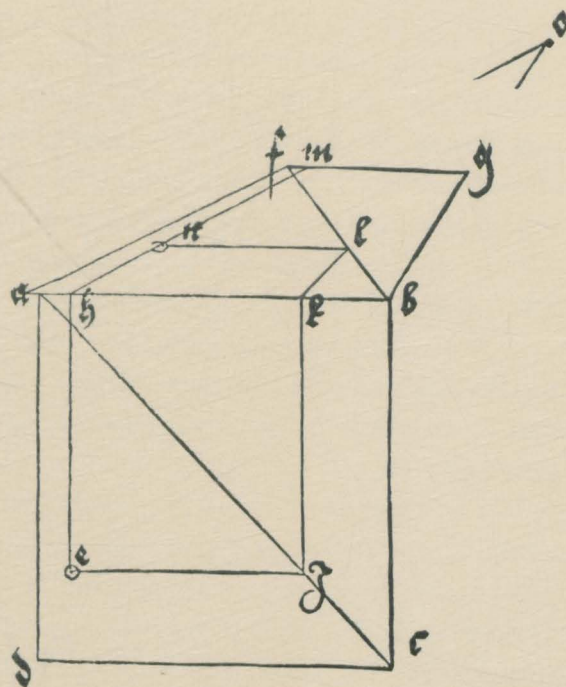


FIG. 37

a logical way, might have produced a point of view much like that utilized on occasion by such a modern master as Degas, but Dürer failed to be logical in his utilization of the high center point. This came about because of his failure to understand that when a picture is made, everything in it, architecture, objects, and figures alike, needs to be represented from one given point of view if the picture is to have any spatial homogeneity.

Analysis of the prints in the *Life of the Virgin*, including those made both before and after Dürer's Italian trip of 1506-1507, shows that while he habitually used a low center

point in sketching his single figures, it was also his habit to use a high center point in sketching or inventing his architecture. From this we learn how he went about building up his compositions. After he had finished his several sketches of the individual figures and of the architecture for a print he copied these sketches off pedantically onto his block — his architectural setting with its high center point and within that architectural setting, wherever he wanted them, his figures, each with its own particular low center point — in such a manner that his obvious, and so to say official, architectural center point had nothing whatever to do with his figures and their various undefined and low center points. The result was that although the architecture and each of the figures was possibly correct from its own special point of view, all but one of them was sadly incorrect from any single point of view.

We find this same lack of pictorial internal cohesiveness in Dürer's great show piece of perspective rendering, his engraving of Saint Jerome in His Study (B. 60). If, in working out this picture, Dürer had followed the simple rules of the game as laid down by either Alberti or Viator, he would not have got himself involved in absurdity after absurdity. The top of the saint's table is of the oddest trapezoidal shape — certainly it is not rectangular. Neither is it level with the floor under it. Moreover, the floor itself is not flat, for somewhere between the table and the bench at its right it takes a sudden tilt and slides off in a new direction. The bench, if a correct projection of it were actually to look like Dürer's picture of it, would have a shape that would astonish everyone, including Dürer himself. These oddities of shape were as carefully disguised or camouflaged by shading as was possible, but anyone who cares to rule lines on a

photograph or reproduction of the engraving will find these and many more to keep them company.

Our analysis of Dürer's perspective theory and practice may perhaps help us to understand how it happens that, no matter what the apparent simplicity and straightforwardness of any of his pictures may be, we are always aware that it contains or is based upon an elusive and tantalizing contradiction. We have seen that his formal perspective construction (the *nahere weg*) contains logical contradictions. It is obvious from his prints (e.g. the Saint Jerome in His Study, B. 60) that he did not understand that parallel lines in parallel planes meet on the axis of intersection of those planes — a fact that follows immediately out of Alberti's and Viator's constructions. His studies of the proportions of the human body were not based upon anatomy (i.e. upon interior relations) but upon visual shapes in standardized locations (i.e. upon exterior relations). Changes in the locations of his figures within the emphasized and most obvious perspective space of their architectural settings, that is, in their external relationships, were not accompanied by the transformations of their visual (or perspective) shapes that are logically required if their interior relations are to remain invariant through changes in location. These distortions were coupled with the utmost realism in the delineation of forms as seen in locations which they but rarely occupy in his finished pictures. The consistency with which he carried out these various distortions amounts almost to a methodical denial of the homogeneity of space. This fundamental contradiction of one of the great intuitive bases of experience produces a subtle psychological malaise in the beholder of his work that, not being readily traceable to an obvious cause, is

doubtless one of the principal reasons for the peculiar fascination that his work has always exercised over the minds of men. It may also be that this basic contradiction is responsible for the fact that so many students of Dürer's work seem always to be working at some conundrum which, like the squaring of the circle, is incapable of solution.²²

(22) Just as it did not seem necessary to discuss Alberti's famous perspective net so it does not seem necessary to discuss Dürer's even more famous but wholly impractical mechanical aids to perspective. In each instance the basic theory is the important thing and that is set forth in the practical geometrical construction. If the present writer's memory is correct, for it is not a matter to waste time in verifying, the writers who have made most of the nets and other contraptions have also been those who have come to greatest grief in their attempts to explain the geometrical constructions.

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APPENDIX

FACSIMILE OF THE FRENCH TEXT OF VIATOR'S
DE ARTIFICIALI PERSPECTIVA

TOUL, 1505

Leuure deuant mis parfait/et veu par aucuns: leur a semble estre reqs le transcrire et interpreter en vulgar: affin que les non clers/ puissent aussi entendre le contenu: Et oultre/ que plusieurs diront/ que les personnes (qui ne sont que grossiemēt efformees) deussent auoir estre entierement pourtraites: Surquoy/ au premier point/ pour la cause dite/ est satisfait cy apres: Et quant au scō touchant les personnes/ est ass auoir/ que ainsi ont este faites et laissees en bosse ou similitude tant seulemēt/ de industrie et certai ppos. pour plus facile comprehension de lartifice: car lintētion du liure nest/ que de adapter es edifices ou espaces designez en icellui/ ou autres telz que lon volra/ les grandeurs & minozations en distance desd personnes: (cōme il est touchie en larticle Droinde/ vers le meillieu.) et nō point de former particulièrement. lesd psonnes. Laquelle formatiō/ et ceste partie de perspectiue/ ont diuerses consideratiōs/ successiuemēt et lune apres lautre itelligibles: (dōt ceste doit estre premier cōceue. Et par ainsi (des querās les p̄icipes de la matiere (car les doctz et expers ne sont a enseigner/ mais fait a aprendre deulx) cue labitude de la maniere proposee la poesie et designation des lineamens desd personnes/ telles ou autres quil plaira/ se peut tousiours comprendre: faire et exprimer/ quant il plaist Laquelle consiste en dimēsiōs arismetrales et doctrine desd maistres (sicōme autres secrez de lart pictozale (dōt les italz tiēnent la palme) avec exercice de veue actuel/ & obfiguratiō parmeluree/ ainsi quil est contenu on deuantdit article/ peu deuant la fin. Science de argutz et ingenieur entendemens/ que les grans & haulx esperis ont/ tousiours (et meritement) amplecte et magnifie/ ensemble les parfaiz artifics dicelle: representans les choses passees & absentes/ cōme instantes et presentes: et veues/ cognoscibles au premier regart: telles qua retenir les speculars: esleuer et mouuoir corages a vertu et diuine action/ admiration/ et benediction: solacier et releuer les ēnuyz de la vie humaine/ et autres choses desirables faire & exprimer: ffinalment (pour reprendre le propos) amables & extimables/ cōme aussi touz perspecteurs et operateurs de choses excellentes et dignes.

*De perspectiua positina compendii
Abregie de perspectiue positine.*

Fundamentis: Quant on veult edifier on cōmence aux fondemens: En apres/ on dresse ou bastist les murailles et corps de ledifice: Et consequemēt/ on appose le toict et couuerture. Ainsi cōuient faire en la presente matiere: Il fault premier auoir cognoissance des p̄icipes: Et iceulx entenduz/ soy appliq̄

a la maniere de proceder a besoigner. Et finalement se exercer a pourtraire et figurer artificialmēt les choses veues ou conceues. Quant aux principes/ il est perspect et deduit par speculations de philosophes/ que toutes choses sont veues cōme par lignes procedēs de lueil/ Cest assauoir par le triāgle: du quel labasse est la chose veue: & son diametre discourt/ par la motion de lueil/ sur les parties dicelle chose veue: Touttefois lumiere ne ist pas de lueil: mais la clarte exteriorē cheant en icellui/ reflecte/ comme dun miroir ardent: par quelle reflection les formes des choses sont conceues & apprehendees. Desquelles formes/ la designatiue et figuratiue expression/ est deriuee du point: lequel (cōbien que soit indiuidu) est euolue en ligne et en lignes: dont figures sont composees/ par lesquelles/ avec points/ et telles lignes/ la voye au propos est ouuerte.

Punctus principalis: Le point principal en perspective doit estre constitue et assis au nyueau de lueil: lequel poit est appelle fix/ ou subiect. En apres/ vne ligne produite et tiree des deux pars dudit point: et en icelle ligne doiuent estre signez deux autres points/ equedistans du subiect: plus prochains en presente/ et plus esloignez en distant veue: lesquels sont appelez tiers points. Et en icelle ligne peut on faire autres poits/ ou il est herra apprest de edifice de plusieurs angles/ ou autre chose de diuerse situatiō. Laquelle ligne est appelee piramidale: car les angles ou pointes des piramides (dont cy apres sera dit) sont deduiz des points en icelle constituez. Et est aussi appelee orizontale: car elle mōstre le soleil ouēt/ et le absconde occident: et tousiours adēque en pareille haulteur lueil de lōme/ ou quil monte & se trāsporte: soit a la summite de haulte tour/ ou du plus esleue mont qui soit. A quelle haulteur la extremite et de terre/ et de mer/ se doit tousiours terminer: se plus haults monts/ entremoyens/ ne sont obiectz.

DEinde: En apres/ vne autre ligne plus basse/ se doit asseoir/ qui sappelle ligne terre: Et en icelle (se on pretend sureriger edifice/ ou cōceur raison de quelque dimension) disposer avec le compas/ aptemēt ouuert/ poits partiz: plus/ ou moins: selon l'opportunitē du propos conceu. Et en besoignāt fault vser d'autres points/ necessaires a l'apprest des choses: qui ne sont pas cy designez: mais seront manifestez par les figures.

Pures quoq: Et plusieurs autres lignes sont a tirer: Desquelles toutes yllans des poits cōstituez en la ligne piramidale/ generalmēt sōt appelees radiales. car elles procedent diceulx points/ comme les raiz procedent du soleil ou des estoilles. Specialment touttefois/ les lignes laterales des piramides/ sont

nōmes visuales: denotās le subiect en perspectiue: et les moiennes dicelles sapel-
lent diametrales/ cest adire les diuisans en deux pties equales. Quāt aux lignes
seruans aux assemblemens et adunations des edifices/soient directes ou perpen-
diculaires/elles nont point autres noms.

Figurarum autem: Quāt aux figures/les vnes sont mises pour elemēts/
les autres pour exēples/ou inductiues de besoigner en plus grandes. Mais
cy est faite mencion de celles qui sont mises pour elements. Toutes lesquelles/
procedent de la spherique: Car le triangle/et le quadrāgle (qui principalmēt seruēt
a la perspectiue) sont deduiꝝ et iustificiez par icelle. Duquel triangle procedent les
pyramides: qui ont similitude de flāme de feu: large par bas/et acue en hault: & se
peuent toutes et chūnes lesd pyramides/ faire plus larges/ou plus estroites: plus
longues/ou plus breues: et autres figurer/pour et selon lexigeuce des euuers quō
emprendra: Car a la gisant a dextre/le fait la contraire a la senestre: & semblable-
mēt a la pendente. Et toutes pyramides/en quelque maniere que soiēt faites/cō-
courent/ & besoignent ensemble/cōme les lettres: et leurs angles ou pointes nais-
sent des points assignez en la ligne pyramidale: Excepte la pointe de la pyramide
signee on penultime lieu: laq̄lle est deduite dun point coniect en lair. Et sil fault
dailleurs deduire autres pyramides/ ce enseignera la speculation des choses quō
boldra faindre: La premiere figure/sapelle trigone ou triāgle: la seconde/pyrami-
de droite: la tierce/euerse: la quarte/gisant: la quinte/double: la sexte/diffuse: la
septime/bicozne ou coznue: la octaue/pendente: la neuvieme/aereale: la dixieme/
tetragone ou tetriangle: Par lequel les espaces depingendes/ sont a disposer: ou
par pauement limite: ou (se aux choses quon boldra faindre/ou a lordonnance di-
celles/il sera necessaire) par distances discrettes.

Queterum. En oultre/la diuersite des regars des choses obiettes est touſ-
iours a cōsiderer: mesmement de edifices. Car on les voit de front/ou par
l'angle. Cest adire par deuant, ou par le coign. & les peut on veoir equilateralimēt
ou inequilateralimēt: et de siege comun/ou elleue. et (cōme il a este touchie dessus)
de presente/ ou distante veue. Et en statuēt ou assiant psonnes esd edifices/ faut
adequer ou conformer leur quantite a la magnitude diceulx: affin q̄ toutes choses
aptement cōuiennēt/et redolent ou representent industrie artificiale.

Ad huc notandum est: Encoz est a noter que chose on magnitude visible
(qui est la matiere de cest art) est aucunefoiz regardee pour estre conceue et

comprise entierement: et lors le diametre de la pyramide ou triangle visual/ chet droitement sur icelle chose: Aucunesfoiz est regardee/ pour que lune partie soit plus attentement intuite et parueue: et lors le diametre de la pyramide est couverti a icelle partie: cōme le guille dun horologe dit quadrant/ se pierre daymant est eircumduit. Duquel ppos l'exemple appert par le tetragone cy apres premier mis: erect & elleue: lequel qui tout ensemble regardera/ du premier propos largument et cōceptiō tiendra: Et qui au corne ou angle dextre flectera lueil/ a icelle part/ la vertu visuelle trāsportera le diametre de la pyramide. Et q̄ au fenestre/ peillemēt.

Minoratio autem: La minoration du tetragone/ cest adire du quadrangle/ sterne et couchie/ ou gisant en plain/ se comprēt par les lignes visuales che/ ans sur les angles dicellui inferiores et pchāins: et par les sectiōs des diametres des pyramides inclinees/ protendues des tiers poits esloignez du subiect/ au double des precedēs: on voirciēt plus/ ou aussi moins que le double/ selon le siege du fignēt/ et presente ou distante veue: et le sercle qui entour le quadrangle erect se demōstre speric/ est a l'entour du sterne et gisant/ oual/ ou lenticulaire: selon la difference des sieges & veues premis. Desquelles propositions se peuvent comprendre sur la seconde figure. La tierce consequēte figure/ contient le quadrangle sans cir/ cumference sperale: demorans les pyramides premises: Desquelles touteffoiz es autres ensuiuātes figures ne seront pas entieremēt mises: mais ce seulement dicelles que sera necessaire. Combien que tousiours soiēt sobz faintes ou sobz entēdues: cōme il apperra a ceulx qui y regarderont.

Diminutio quoq; pauimenti: Et la diminution du pauemēt/ se prent sur le quadrangle/ parti de points/ et distinct de lignes radiales/ aux sections des diametres des pyramides inclinees (cōme il est deuant touchie) et plus a plain en maintes manieres demonstre/ par les quatre quinte et sexte figures/ avec les quatre sequentes: & se peuvent faire autres diuers pauemēs/ et multiplier & dilater au plaisir de l'ouurier. Et qui veult limiter les distāces despace chāpestre/ en pour/ traitant les droites lignes de pauement sobz faint/ aura son intencion: se par autre geometrale industrie n'aura cogneu le faire. Et quant a la minoration des pson/ nes (dōt les plus esloignēz apperent a lueil/ moindres que les prochaines) elle est prinse en deux manieres. Car on les puet regarder de siege commun/ ou de siege elleue. Se on les voit de siege cōmun. on prēt leur haulteur et minoration/ de leurs piez/ a la ligne pyramidale: laquelle ligne/ les psonnes puēt excéder des yeux en sus: ou plus/ en procere et giganteale magnitude: Se on les voit de siege elleue/

fault vser de la piramide inclinée/ yssant du point subget/ ou de lun des tiers: de la
 quelle/ la latitude basse/ soit ouuerte a la haulteur de la prochaine et plus grande
 psonne qu'on voldra statuer en l'espace. selon la inaretation de laquelle piramide/
 lesd psonnes prendrôt de la plante au hault des testes leur minoration. Desquel-
 les trois ppositions (cest assauoir de limiter espaces champestres: et de asseoir et
 minorer les quantitez de psonnes veues de siege cōmun/ et aussi de siege esleue)
 les exemples apperent par les trois figures supuans les pauemens. Dont/
 la derreniere est ordonnee/ selon siege peu esleue/ et se puet esleuer plus hault/ et
 encozes surelleuer/ a la ligne orizontale/ et aussi la pointe de la piramide inclinée/
 ensemble esleuer/ prolonger/ ou dilater: ainsi quil est deuant note en l'article des
 figures elementaires: cōme avec ce sera cōprinz des intens speculateurs. Et en
 quelque lieu que lon volra faindre enfans ou adolescens/ sera leur quātite a pren-
 dre selon leurs aages: eu regart a la quantite des plus grandes psonnes: ausquelz
 enfans/ a autres psonnes/ ou que soient assignees/ pportiōs legitimes serōt don-
 nees selon leurs magnitudes. Et ce que par tout est dit des psonnes/ est entendu
 semblablement des autres animants. / Et au regart de la diminution de la latitude
 et grosseur de colūnes et semblables choses/ elle se prent par la dimnntiō du tetra-
 gone: en surelleuāt/ cōme de la basse/ lesd colūnes a choses: et les haulteurs dicelles/
 se prenēt par piramides cōuenables/ Cest assauoir par les gisantes/ ou pendētes:
 selon la sublimite desd colūnes/ et le siege du voiant icelles. Mais il est a auertir/
 que en veue moult distante et esloignee/ est expedient vser souuent effoiz de indu-
 strie et engin: et semblablement/ en maintes autres choses/ laisses a la speculatiō
 des plus argutz et subtilz.

DRoinde: Consequenimēt (Des choses dessus dites/ a leffect pretendu plai-
 nement prises) les figures exēplaires/ descriptes a inductiue/ soiēt veues:
 desquelles aucunes sont erigees de la memoire de edifices ou peintures aucunes/
 foiz et pieca veues: les autres/ de la speculation du ditfant. Desquelles seront cō-
 ceues par les elemēs premis: cōme sont parolls par les lettres. Du m'x/ sur icel/
 les figures/ seront entenduz les offices diceulx elemēs: cōme ilz ont aptitude ma-
 nuductiue a statuer les plaines et plates formes/ dont sont esleuez lesd edifices et
 choses effingibles/ a a leurs assemblēmēs et adunatiōs. Car es figures veues de
 plaine frōt/ la piramide droite/ constitue le pauemēt: a deux ou plusieurs gisantes/
 les costez ou paroiz: la euerse/ la volte ou foict dessus: Et es figures veues angu-
 lairement/ la diffuse et la cornue/ besoignēt: la double/ en toutes deux: la pendente/
 sert a plusieurs choses: la aereale/ a faindre aucuns degrez: Le surplus sera veu

par l'expérience dicelles figures. Et quelles toutefois les proportions particulieres des personnes ne sont pas précisément obseruees: car cest vne autre maxime speculation/dont cy n'est aucunement touchie: mais les quantitez de leurs grandeurs signāment/ qui font au propos: et les edifices ne sont pas pourtraiz ne varieez ou enrechiz de fueillages et diuers signes/ a toute plenitude/ (car maintes choses se mettent en grans volumes/qui sont demorees) pour les petiz espaces/ et mesme de industrie ny ont este mis que les principaulx traiz/pour plus facile cōception de lart et de leure. Lequel euure n'est de main de paître mais de qui aime les paîtres et artisans/et touz viuāns: qui aux queraus les principes de la perspectiue de l'uld/ quelz il a peu consuyz des liures/ euures/ a ozacles/ ou collatiōs/ de tresperitz/ passant par les fins de lozraine a volu et este curieux mettre par escript. Dultre les quelz principes/maints secretz sont de la noble subtilite de peinture/a querir par longue erudition de plusdoctz et maistres en la science dite/ et par actuel contuit ou regart/ avec parmesuree obfiguration et contrefacture des choses uaturelles et artificiales. Ausourplus/les obmissions ou choses moins explozees des sculpteurs' non encozes tritz ou frotez) et du pourtraiant percurrent la sixieme decade vueillēt dresser les voians: parfaisans le tout au piuseau doux et gracieux. A la loenge du souuerain artific dieu/ pere/ filz/ et saint espit Qui a perspicer et contempler la cite roiale de la souueraine maieste/ vueille conduire touz pacificz viateurs de la terre: en la quelle cōme noz peres sōmes estraingiers et pelerins:

Pro cunctis orat.

Cellui qui a ce liure fait
 Die pour touz de cueur pfaît
 Et supplie tres humblement
 Dier pour lui pareillement

HAbes optime lector: O tresbon lecteur/ present as trefacile abregie de la
perspectiue positifue/ acteste en plusieurs manieres de figures et exemples.
Duquel ne requier grauite ou aornemēt de paroles/ mais plus le sens au fin pre/
pose de la gloire de dieu p̄ice des artifics. Toutelsoiz a plusgrāt declaration des
choles precedētes: est a adiouster/ car es premis articles Adhuc: Minoratio: Di/
minutio: esquelz est faite mencion des figures/ entens de celles qui supuent indi/
stāment la lettre/ es quatre derreniers fueilletz du premier quaterne. Et en l'ar/
ticle Droinde (qui est le dernier) ou est touchie desd figures/ est a entendre de celles
qui sont cōtenues es quatre quaternes ensuyuans/ signez B: C: D: E: desquel/
les/ celles qui sont contenues ou fueillet B. i. sont demonstratiues de eriger edifi/
fices sur leurs plates formes: les aults desd quatre quaternes/ ont leurs speciales
differences/ et en p̄tie ont este imprimees cōme elles sont venues es mais des ou/
riers: affin quō ne pense ordre estre necessaire entre icelles. Dultreplus/ la rigle
que le liure appelle norma/ les artisans des gaules dient Niveau. Et les formes
que led liure nomme plaines/ lesd artisans appellent plates formes: Lesquelles
plates formes sont cōmencees et dressees par les pauemens aptement pourtraiz/
aduōbrez et considerez sur iceulx les espaces & distances ou mesures opportunes.
Enquoy faisant est requise intente speculacōn. figure ouale est oblongue/ a la
semblāce dun oeuf. figure lenticulaire/ approche plus de la sperale: toutelsoiz n'est
pas ronde. Section est où vne ligne trespasse lautre.

finis. Laus deo.

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